Studies in Nonlinear Dynamics & Econometrics

Volume 5, *Issue* 4 2001 *Article* 3

Microeconomic Models for Long Memory in the Volatility of Financial Time Series

Alan Kirman [*] Gilles Teyssière [†]

[*]Institut Universitaire de France & GREQAM

[†]GREQAM & CORE

http://www.bepress.com/snde

ISSN: 1558-3708

This volume was previously published by MIT Press.

Microeconomic Models for Long Memory in the Volatility of Financial Time Series

Alan Kirman
EHESS, Université d'Aix-Marseille III
Institut Universitaire de France & GREQAM
kirman@ehess.cnrs-mrs.fr

Gilles Teyssière
GREQAM & CORE
gilles@ehess.cnrs-mrs.fr

Abstract. *We show that a class of microeconomic behavioral models with interacting agents, derived from Kirman (1991) and Kirman (1993), can replicate the empirical long-memory properties of the two first-conditional moments of financial time series. The essence of these models is that the forecasts and thus the desired trades of the individuals in the markets are influenced, directly or indirectly, by those of the other participants. These "field effects" generate "herding" behavior that affects the structure of the asset price dynamics. The series of returns generated by these models display the same empirical properties as financial returns: returns are $I(0)$, the series of absolute and squared returns display strong dependence, and the series of absolute returns do not display a trend. Furthermore, this class of models is able to replicate the common long-memory properties in the volatility and covolatility of financial time series revealed by Teyssière (1997, 1998a). These properties are investigated by using various model-independent tests and estimators, that is, semiparametric and nonparametric, introduced by Lo (1991), Kwiatkowski et al. (1992), Robinson (1995), Lobato and Robinson (1998), and Giraitis et al. (2000, forthcoming). The relative performance of these tests and estimators for long memory in a nonstandard data-generating process is then assessed.*

Keywords. long memory, microeconomic models, field effects, semiparametric tests, conditional heteroskedasticity

Acknowledgments. We are greatly indebted to Hans Föllmer, with whom the stochastic process used in this paper has been derived, and to Raj Bhansali, Russell Davidson, Liudas Giraitis, Clive Granger, Marc Henry, Cars Hommes, Piotr Kokoszka, Remigijus Leipus, Donatas Surgailis, and Bas Werker for useful comments and discussions. We thank the editor, Bruce Mizrach, for comments that have improved the presentation, and seminar participants in the Weierstrass Institute of Applied Analysis and Stochastics in Berlin, October 1998; the Department of Probability and Mathematical Statistics of Charles University in Prague, November 1998; the Division of Statistics in the Department of Mathematics of the University of Liverpool, February 1999; the 1999 Econometric Society European Meeting in Santiago de Compostela, August 1999; the Society for Nonlinear Dynamics and Econometrics Annual Symposium in Atlanta, March 2000; the CeNDEF seminar at the Free

University of Amsterdam, February 2001; the Eighth EWGFM Meeting in Vilnius, May 2001, for comments. We also thank Olsen & Associates and Rakhal Dave for providing some of the data used in this paper. Both authors will blame each other for the remaining mistakes.

1 Introduction

Over time, a clearer picture of some of the statistical features of time series of asset prices has emerged. An extensive literature has been developed on the subject of testing for these features. Few models based on microeconomic behavior that actually generate these characteristics have been proposed, however. This paper presents a class of models in which agents interact stochastically on a financial market and which do, in fact, generate the statistical characteristics found empirically.

The dynamics of the two first-conditional moments of asset prices are rather complex. The mean series are, in general, nonstationary, have unit roots at daily frequency, and exhibit bubbles, with a clustering of large deviations, whereas the volatility series display a form of significant dependence between very distant observations called long-range dependence or long memory (see Mandelbrot 1963, Taylor 1986, and Granger and Ding 1995, 1996).

The econometric literature has focused on the statistical tests and models for testing and representing these empirical features. There is a substantial literature on unit roots, nonstationarity tests, and their refinements (see Dickey and Fuller 1979, Sowell 1990, Robinson 1991, 1994a, 1994b, Kwiatkowski et al. 1992, MacKinnon 1994, 1996, and Lee and Schmidt 1996, among others). Basic references for testing for bubbles are Blanchard and Watson 1982, Hamilton 1989, and Evans 1991.

As a consequence of the martingale property of daily prices P_t, the log of returns $R_t = \ln(P_t/P_{t-1})$ is uncorrelated. However its power transformation $|R_t|^\delta$, where δ is a positive real number, is characterized by a long-range serial dependence, or long memory. This long-range dependence in the volatility can be modeled by resorting to the class of long-memory autoregressive conditional heteroskedasticity (ARCH) processes, introduced by Robinson (1991) (see Ding and Granger 1996, Granger and Ding 1995, and Giraitis, Robinson, and Surgailis 2000), the long-memory stochastic volatility models (see Harvey 1998, Breidt, Crato, and de Lima 1998, and Robinson 2001), the multifactors models of Gallant, Hsu, and Tauchen (1998), and the multifractal model of asset returns developed by Mandelbrot, Fisher, and Calvet (1997).

The fact that the class of fractional models can capture the rich dynamics of asset prices and their transformations is not particularly surprising, as fractals mathematics has been used for describing a large variety of complex phenomena in geophysical sciences, economics, engineering, etc. A simple descriptive approach is incomplete, however, as many empirical results show some regularity in the long-memory structure across economic series: Teyssière (1997, 1998a) revealed that some daily and intraday foreign exchange (FX) rates returns display the same degree of long memory in their conditional variances and covariances. Since these regularities are not fortuitous and are presumably caused by some economic phenomenon, the obvious next step is to devise a structural model generating these statistical properties.

Structural models for long memory are rather rare. Willinger, Paxson, and Taqqu (1998) have proposed a framework explaining the self-similarity in the mean series of Ethernet traffic. Box-Steffenmaier and Smith (1996) and Byers, Davidson, and Peel (1997) have considered aggregate popularity models explaining persistence in opinion polls. The results of this series of contributions are derived by making some distributional assumption on the parameters of the models and by resorting to statistical distributions that when aggregated lead to long-memory processes. The long-memory properties of the Mandelbrot, Fisher, and Calvet (1997) multifractal model rely on the multifractal distribution of the trading time.

Our approach is based on behavioral models that do not resort to the above-mentioned statistical distributions. The essence of these models is that the forecasts and thus the desired trades of the individuals in the markets are influenced, directly or indirectly, by those of the other participants. This interdependence, called a "field effect,"[1] generates "herding" behavior that affects the structure of the asset price dynamics. This research was motivated by the fact that one of the authors (the econometrician) has been working on the long-memory properties of financial data and was embarrassed by the lack of theoretical models explaining his empirical findings. He looked at one of the models devised by the other author (the microeconomist) in 1991. After a few changes, a convincing answer to his questions emerged.

These models replicate other empirical properties of financial data. For large samples, the simulated series of returns can be fitted by integrated generalized ARCH (IGARCH) models (see Kirman and Teyssière 2001, forthcoming). Furthermore, like absolute returns on asset prices, our simulated series of absolute returns do not display a trend and then differ from standard long-memory processes.

These models were originally intended to explain bubbles in asset price series and to develop an economic model to reinforce Evans' (1991) explanation of the possibility of "rational bubbles." A number of models generating herding behavior have been proposed by Day and Huang (1990), Banerjee (1992), Bikhchandani, Hirshleifer, and Welsh (1992), and Welch (1992), for example. The models proposed here neither rely on persistent erroneous beliefs nor emphasize convergence to a single self-fulfilling belief, as in the "informational cascades" or "sunspots" literature (see, e.g., Woodford 1990). Our models exhibit continual switching from one dominant belief to another, and there is no convergence to a particular state. The appropriate equilibrium notion is that of a limiting distribution.

The paper is organized as follows: in section 2 we give some empirical evidence that leads us to consider structural models for long memory. Given that the behavioral models considered here do not belong to a standard family of stochastic processes, we consider only model-independent tests and estimators for long-range dependence, which are presented in section 3. Section 4 presents these behavioral models. Simulation results are reported in section 5, in which the relative performance of these semiparametric and nonparametric tests and estimators with respect to a nonstandard data generating process is discussed. Section 6 concludes.

2 The Empirical Properties of Financial Data

2.1 Long-memory processes

A stationary process Y_t is called a stationary process with long memory if its autocorrelation function (ACF), $\rho(k)$, has asymptotically the following hyperbolic rate of decay (see Beran 1994, Granger 1980, Granger and Joyeux 1980, Hosking 1981, and Robinson 1994a):

$$\rho(k) \sim L(k)k^{2d-1} \quad \text{as } k \to \infty \tag{2.1}$$

where $L(k)$ is a slowly varying function[2] and $d \in (0, 1/2)$ is the parameter that governs the slow rate of decay of the ACF and then parsimoniously summarizes the degree of long-range dependence of the series. In contrast, the ACF of a "short-memory" process, such as an autoregressive moving average (ARMA) process, converges at an exponential rate (i.e., very quickly) to zero.

Equivalently, the spectrum $f(\lambda)$ of a stationary process with long-memory parameter d can be approximated in the neighborhood of the zero frequency as

$$f(\lambda) \sim C\lambda^{-2d} \quad \text{as } \lambda \to 0^+ \tag{2.2}$$

where C is a finite, strictly positive constant. The autocorrelations of a long-memory process are then not summable (i.e., $\sum_{k=1}^{\infty} \rho(k) = \infty$), and the spectrum of a long-memory process has a pole at frequency zero as $\lim_{\lambda \to 0^+} f(\lambda) = \infty$.

[1] See Aoki (1996) and Durlauf (1997) for surveys on models with interacting agents.

[2] A function $L(k)$, $k \geq 0$, is called a "slowly varying function" if $L(\lambda k)/L(k) \to 1$ as $k \to \infty$, $\forall \lambda > 0$.

We detect long memory and estimate the long-memory parameter d in various ways. In a first approach, we consider a class of parametric long-memory models, the simplest one being the fractional noise or $I(d)$ process, defined by

$$(1-L)^d y_t = \varepsilon_t, \quad \varepsilon_t \sim i.i.d.\ (0, \sigma^2), \quad (1-L)^d = \sum_{k=0}^{\infty} \psi_k L^k, \quad \psi_0 = 1, \psi_k = \prod_{j=1}^{k} \left(1 - \frac{1+d}{j}\right) \tag{2.3}$$

where the real parameter d is called the fractional degree of integration of the process and $(1-L)^d$ is the fractional difference operator. This process is stationary if $d < 1/2$, mean reverting if $d < 1$, and invertible if $d > -1$ (see Odaki 1993).

Daily prices P_t are normally found to be $I(1)$, which is a consequence of the efficient-market hypothesis; that is, $E(P_{t+1} \mid I_t) = P_t$, where I_t denotes the information set available at time t. Although the logs of returns $R_t = \ln(P_t/P_{t-1})$ are then $I(0)$, their power transformation $|R_t|^\delta$ displays long memory. Figure 1 displays the autocorrelation function (ACF) of the absolute returns on pound–U.S. dollar for 1979–97. The slow decay of this ACF is typical of a long-memory process and suggests the presence of long memory in the volatility of asset returns.

Long-range dependence in the conditional second moments was first considered by Robinson (1991), who introduced the class of long-memory ARCH processes for testing for no ARCH effects. The general form for an ARCH(∞) is

$$R_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim D(0,1) \quad \text{with} \quad \sigma_t^2 = \sigma^2 + \sum_{j=1}^{\infty} \psi_j (g(\varepsilon_{t-j}) - \sigma^2) \tag{2.4}$$

for some $\sigma^2 > 0$, where $D(0, 1)$ is a distribution with zero mean and variance equal to one, and $\sigma^2 - g(\varepsilon_t)$ is a martingale difference. If $\psi_j = 0$ for $j > p$, this model is an ARCH(p) model, while if the infinite sequence $\{\psi_j\}_{j=1}^{\infty}$ has the asymptotic hyperbolic rate of decay $\psi_j = O(j^{-(1+d)})$, then equation (2.4) defines an ARCH(∞) process with long-memory parameter d: for this process, shocks to the error terms have a persistent effect on the conditional variance.

The class of long-memory ARCH models is able to capture the hyperbolic rate of decay of the volatility autocorrelations, provided that a suitable model is selected. Figure 2 displays the empirical ACF of the absolute returns $|R_t|$ on the S&P 500 Composite Index P_t, with the averaged ACF of 2,000 simulated fractionally integrated nonlinear generalized ARCH (FINGARCH)-t processes estimated on these data.[3] The conditional heteroskedastic function of a FINGARCH process is defined as

$$\sigma_t^\delta = \frac{\omega}{1-\beta(1)} + \left(1 - \frac{(1-\phi(L))(1-L)^d}{1-\beta(L)}\right) |\varepsilon_t + \gamma \sigma_t|^\delta \tag{2.5}$$

Although this model is able to capture the hyperbolic decay of the ACF more adequately than most other long-memory ARCH models,[4] it shares the same drawbacks as the long-memory volatility models: the asymptotic normality and root-n consistency of the approximate maximum likelihood estimator are only conjectured on the basis of Monte Carlo simulations, and there might be no strictly stationary solution to the equations defining a fractionally integrated GARCH process (see Giraitis, Kokoszka and Leipus 2000, Kazakevičius and Leipus 2002, Kazakevičius, Leipus and Viano 2000, and Giraitis and Surgailis 2001, which contradicts the assertion of Baillie, Bollerslev, and Mikkelsen 1996). Furthermore, the unconditional variance of a long-memory practically integrated GARCH type process does not exist, which is not consistent with what is empirically observed for several series of asset returns (see, e.g., Dacorogna et al. 1995).

[3]This picture is borrowed from Teyssière (1998b), where the FINGARCH process was introduced.
[4]Another case of perfect fit is given in Ding and Granger (1996). In other cases, the quality of the fit is not perfect.

Table 1
Estimation of the fractional degree of integration for the series of absolute returns on pound–U.S. dollar $|R_{1,t}|$, Deutsche mark–U.S. dollar $|R_{2,t}|$, French franc–U.S. dollar $|R_{3,t}|$, $\sqrt{|R_{1,t}R_{2,t}|}$, $\sqrt{|R_{1,t}R_{3,t}|}$, $\sqrt{|R_{2,t}R_{3,t}|}$ for the period January 1986–January 1997

m	$\|R_{1,t}\|$	$\|R_{2,t}\|$	$\|R_{3,t}\|$	$\sqrt{\|R_{1,t}R_{2,t}\|}$	$\sqrt{\|R_{1,t}R_{3,t}\|}$	$\sqrt{\|R_{2,t}R_{3,t}\|}$
$[n/4]$	0.2083	0.1682	0.1735	0.1847	0.2014	0.1745
$[n/8]$	0.2641	0.2911	0.2447	0.2777	0.2629	0.2512
$[n/16]$	0.3740	0.3767	0.3674	0.3879	0.3927	0.3682

2.2 Empirical evidence

We observe some regularity in the estimated long-memory components of the volatility. As the volatility of a series R_t can be represented by its absolute value $|R_t|$ (see Granger and Ding 1995), we define the "covolatility" between the series $R_{1,t}$ and $R_{2,t}$ by the expression $\sqrt{|R_{1,t}R_{2,t}|}$. Teyssière (1997) analyzed the volatility and covolatility of daily returns on pound–U.S. dollar and Deutsche mark–U.S. dollar by introducing the bivariate unrestricted long-memory ARCH model, defined as

$$\begin{pmatrix} R_{1,t} \\ R_{2,t} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \quad \text{with} \quad \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} s_{11,t} & s_{12,t} \\ s_{12,t} & s_{22,t} \end{pmatrix}\right] \tag{2.6}$$

the specification of the conditional covariance matrix being either

$$s_{ij,t} = \frac{\omega_{ij}}{1-\beta_{ij}(1)} + \left(1 - \frac{(1-\phi_{ij}L)(1-L)^{d_{ij}}}{1-\beta_{ij}L}\right)\varepsilon_{i,t}\varepsilon_{j,t}, \quad i, j = 1, 2 \tag{2.7}$$

or

$$s_{ij,t} = \sum_{k=1}^{\infty} \frac{B(p_{ij}+k-1, d_{ij}+1)}{B(p_{ij}, d_{ij})}\varepsilon_{i,t-k}\varepsilon_{j,t-k}, \quad i, j = 1, 2 \tag{2.8}$$

i.e., both conditional variances and covariances are modeled as long-memory ARCH processes. Estimation results by quasi-maximum-likelihood methods led us to accept the hypothesis that the three long-memory parameters are the same (i.e., $d_1 = d_2 = d_3 = 0.4187$ for the period 1979–97). This empirical result is confirmed if we estimate the degree of long-range dependence of the volatilities and covolatilities with Robinson's (1995) semiparametric estimator presented in the next section. We also consider in this paper the series of French franc–U.S. dollar daily returns and estimate the degree of long memory of the three volatilities and three covolatilities with the same estimator (see Table 1). As the estimated value of $\hat{d}$ is the same for the different bandwidths m, we can then conclude that the volatilities and covolatilities of these three foreign-exchange (FX) rates share the same degree of long memory.

We also evaluate the degree of long memory of the volatilities and covolatilities of the pound–French franc and Deutsche mark–French franc with the same estimator. Results displayed in Table 2 show that the level of long memory depends on the FX market, however, the property of a common long-memory component is preserved.

These regularities in the long-memory properties are certainly not fortuitous, since such a common degree of long-range dependence has also been observed for more recent series of intraday FX rates (pound–U.S. dollar, Deutsche mark–U.S. dollar, and yen–U.S. dollar. Table 3 reports the estimates of d for the 30-minutes-spaced Olsen & Associates HFDF-96 series of absolute returns on the following FX rates: dollar–Deutsche mark, dollar–yen, dollar–Swiss franc, and dollar–pound for the year 1996.[5] Similar results on

[5]These series are in ϑ-time; that is, the intraday seasonal component has been removed (see Dacorogna et al. 1993). Since these series do not have the same time scale, we cannot evaluate the covolatilities.

Table 2
Estimation of the fractional degree of integration for the series of absolute returns on pound–French franc $|R_{1,t}|$, Deutsche mark–French franc $|R_{2,t}|$, $\sqrt{|R_{1,t}R_{2,t}|}$, for the period January 1986–January 1997

m	$\lvert R_{1,t}\rvert$	$\lvert R_{2,t}\rvert$	$\sqrt{\lvert R_{1,t}R_{2,t}\rvert}$
$[n/4]$	0.1661	0.1729	0.1448
$[n/8]$	0.2116	0.2167	0.1823
$[n/16]$	0.2690	0.2836	0.2489

Table 3
Estimation of the fractional degree of integration for the series of absolute returns on U.S. dollar–Deutsche mark $|R_{DEM,t}|$, U.S. dollar–yen $|R_{YEN,t}|$, U.S. dollar–Swiss franc $|R_{CHF,t}|$, and U.S. dollar–pound $|R_{GBP,t}|$

m	$\lvert R_{DEM,t}\rvert$	$\lvert R_{YEN,t}\rvert$	$\lvert R_{CHF,t}\rvert$	$\lvert R_{GBP,t}\rvert$
$[n/4]$	0.1738	0.1776	0.1961	0.1810
$[n/8]$	0.2076	0.2252	0.2341	0.2138
$[n/16]$	0.2566	0.2450	0.2597	0.2550

the commonality of the long-memory component have been observed with different statistical methods by Henry and Payne (1997) for high-frequency FX rates and by Ray and Tsay (2000) in the U.S. stock market. Teyssière (1998a) estimated a trivariate fractionally integrated ARMA (ARFIMA)-FINGARCH on 30-minutes-spaced FX returns and observed that the conditional variances share the same long-memory component, and the conditional means have a common antipersistent component.

The game of estimating and comparing the degrees of long memory might be endless, as one can always consider other financial assets and estimators. Such an exercise would be of little interest, however, and a more interesting direction is to find the cause of these striking regularities in the long-memory properties, which are presumably the outcome of some common economic phenomenon.

3 Semiparametric and Nonparametric Tests and Estimators

As we are investigating the properties of behavioral microeconomic models that cannot be reduced to a standard stochastic process, we consider here model-independent tests and estimators, that is, a class of tests and estimators that do not require a specific functional form or a particular distributional assumption on the stochastic process generating the data. We also analyze the behavior and relative performance of these tests and estimators with our nonstandard data-generating process. A similar methodology has been followed by Kirman and Teyssière (2001, forthcoming) for analyzing other statistical properties of the behavioral models studied here.

We first consider a family of estimators and tests that are based on the approximation of the spectrum of an $I(d)$ process in a neighborhood of the zero frequency, as given in Equation (2.2). Robinson (1994c, 1995) proposed two semiparametric estimators that are asymptotically normally distributed and robust to conditional heteroskedasticity of general form, including long-memory conditional heteroskedasticity, with the same limiting distribution as in the homoskedastic i.i.d. case.[6] We consider here only the second estimator, as it is more efficient, its asymptotic distribution is independent of d, and a feasible optimal bandwidth for this estimator has been derived by Henry and Robinson (1996). Künsch (1987) suggested estimating the parameter d by replacing the analytical expression of the spectrum in the Whittle approximate Gaussian maximum-likelihood estimator with its approximation given by Equation (2.2). Robinson (1995) developed this idea and established the asymptotic properties of this estimator, obtained by solving the following

[6]The robustness properties have been established by Henry (2001b) and Robinson and Henry (1999), respectively.

minimization problem:

$$\{\hat{C}, \hat{d}\} = \arg\min_{C,d} L(C, d) = \frac{1}{m}\sum_{j=1}^{m}\left\{\ln(C\lambda_j^{-2d}) + \frac{I(\lambda_j)}{C\lambda_j^{-2d}}\right\} \tag{3.1}$$

where $I(\lambda_j)$ is the periodogram. We assume that the approximation (2.2) holds for a degenerate range of m Fourier frequencies $\lambda_j = 2\pi j/n,\ j = 1, \ldots, m \ll [n/2]$, where $[\cdot]$ denotes the integer part operator, bounded by the bandwidth parameter m, which increases with the sample size n but more slowly as $1/m + m/n \to 0$ as $n \to \infty$. Under appropriate conditions, which include the differentiability of the spectrum near the zero frequency and the existence of a moving average representation, the asymptotic distribution of this Gaussian estimator is

$$\sqrt{m}(\hat{d} - d) \sim N\left(0, \frac{1}{4}\right) \tag{3.2}$$

We use here the formula for the data-driven optimal bandwidth proposed by Henry and Robinson (1996) which, as demonstrated by Henry (2001a), is robust to conditional heteroskedasticity of general form, including the long-memory case.

We consider in this paper several model-independent tests for $I(0)$ against fractionally integrated alternatives $I(d)$. Lobato and Robinson's (1998) nonparametric Lagrange multiplier test for $I(0)$ against $I(d)$ alternatives is also based on the approximation (2.2) of the spectrum of a long-memory process. In the univariate case, the t-statistic is defined as

$$t = \sqrt{m}\hat{C}_1/\hat{C}_0 \quad \text{with} \quad \hat{C}_k = m^{-1}\sum_{j=1}^{m}\nu_j^k I(\lambda_j) \quad \text{and} \quad \nu_j = \ln(j) - \frac{1}{m}\sum_{i=1}^{m}\ln(i) \tag{3.3}$$

where m is a bandwidth parameter. Under the null hypothesis of an $I(0)$ time series, the statistic t^2 is $\chi^2(1)$ distributed.

The three other tests for $I(0)$ against $I(d)$ are based on the assumption that under the null hypothesis of $I(0)$, the standardized series of the partial sums of the process $S_k = \sum_{j=1}^{k}(Y_j - \bar{Y}_n)$ satisfies a functional central-limit theorem. What is required is only the existence of an autocorrelation-consistent estimator of the variance. All these tests make use of the Newey and West (1987) heteroskedastic and autocorrelation-consistent (HAC) variance estimator:

$$\hat{\sigma}^2(q) = \hat{\gamma}_0 + 2\sum_{i=1}^{q}\omega_i(q)\hat{\gamma}_i \quad \text{with} \quad \omega_i(q) \equiv 1 - \frac{1}{q+1} \tag{3.4}$$

where the sample autocovariances $\hat{\gamma}_i$ at lag i account for the possible short-range dependence up to the qth order. The main problem with the statistics considered here is the lack of statistical criteria for choosing the truncation order q, although it should logically be related to the degree of autocorrelation and the length of the series.

Lo (1991) modified Hurst's (1951) R/S statistic, based on the range of the partial sum process S_k, by replacing the standard variance estimator with the HAC estimator. This new statistic is robust to short-range dependence and is defined as

$$R/S(q) = \frac{1}{\hat{\sigma}(q)}[\max_{1\le k\le n} S_k - \min_{1\le k\le n} S_k] \tag{3.5}$$

If $q = 0$, Lo's statistic reduces to Hurst's R/S statistic. Under the null hypothesis of no long memory, the statistic $n^{-\frac{1}{2}}R/S$ converges to a distribution equal to the range of a Brownian bridge on the unit interval: $\max_{0\le t\le 1} W^0(t) - \min_{0\le t\le 1} W^0(t)$, where W_t^0 is the Brownian bridge defined as $W^0(t) = W(t) - tW(1)$, $W(t)$ being the standardized Wiener process. Teverovsky, Taqqu, and Willinger (1999) observed that the probability of accepting the null hypothesis of no long-range dependence depends significantly on q and is overestimated whatever the long-memory properties of the data. These authors suggest using this statistic together with other tests and estimators for long-memory analysis.[7]

The KPSS test, proposed by Kwiatkowski, Phillips, Schmidt, and Shin (1992) and Lee and Schmidt (1996), based on the second moment of the process S_k, is defined as

$$KPSS(q) = \frac{1}{n^2\hat{\sigma}^2(q)} \sum_{k=1}^{n} S_k^2 \tag{3.6}$$

Giraitis, Kokoszka, Leipus, and Teyssière (forthcoming) have shown that under the null hypothesis of $I(0)$, this statistic asymptotically converges to a well-defined random variable $U = \int_0^1 (W^0(t))^2\, dt$, where $W^0(t)$ is a Brownian bridge. Furthermore, these authors pointed out that the cumulative distribution function of this statistic has a series expansion, involving parabolic cylinder functions, which converges very quickly.

Giraitis et al. (forthcoming) proposed a centering of the KPSS statistic. This statistic, denoted by V/S, is then based on the variance of the process S_k:

$$V/S(q) = \frac{1}{n^2\hat{\sigma}^2(q)} \left[\sum_{k=1}^{n} S_k^2 - \frac{1}{n} \left(\sum_{k=1}^{n} S_k \right)^2 \right] = n^{-1} \frac{\hat{V}(S_1, \ldots, S_n)}{\hat{\sigma}^2(q)} \tag{3.7}$$

The limiting distribution of this statistic is a well-defined random variable $V = \int_0^1 (W^0(t))^2\, dt - (\int_0^1 W^0(t)\, dt)^2$, the distribution of which is linked to the distribution of the Kolmogorov statistic by a single change of variable. This statistic has uniformly higher power than the KPSS statistic and is less sensitive to the choice of the order q than Lo's statistic (see Giraitis et al. forthcoming and Kirman and Teyssière forthcoming). Giraitis et al. (forthcoming) have also shown that this statistic can be used in the detection of long memory in volatility for the class of ARCH(∞) processes.

We can estimate semiparametrically the degree of long memory in volatility in the framework of the long-memory linear ARCH model developed by Giraitis, Robinson, and Surgailis (2000), defined as

$$R_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim D(0, 1) \quad \text{with} \quad \sigma_t = \alpha + \sum_{j=1}^{\infty} \psi_j R_{t-j} \tag{3.8}$$

where the infinite sequence of coefficients $\{\psi_j\}_{j=1}^{\infty}$ has the rate of decay $\psi_j = O(j^{d-1})$, with $d \in (0, 1/2)$. Giraitis, Robinson, and Surgailis (2000) have shown that under the condition $L(E\varepsilon_0^4)^{1/2} \sum_{j=1}^{\infty} \psi_j^2 < 1$, where $L = 7$ for the Gaussian case and $L = 11$ in other cases, there exists a stationary solution to the two previous equations, such that the sequence of squares $\{R_t^2\}_{t=1}^{\infty}$ has a covariance function the rate of decay of which is given by Equation (2.1).

Giraitis et al. (2000) have proposed several semiparametric estimators for the degree of long memory of the linear long-memory ARCH process, from its sequence of squares $\{R_t^2\}_{t=1}^{n}$. One of them is the standard R/S "pox-plot" analysis advocated by Mandelbrot and Wallis (1969) and Mandelbrot and Taqqu (1979), among others. The two other estimators are new and simply extend the "pox-plot" analysis to the KPSS and V/S statistics. Interested readers are referred to Beran (1994) and Giraitis et al. (2000) for the technical details of the implementation of the "pox-plot" methods.

4 Microeconomic Models for Long Memory in the Volatility

The microeconomic models used here are derived from Kirman (1991) and Kirman (1993). Their basic foundation is the existence of two groups of agents, called chartists and fundamentalists, who differ in regard

[7] When q increases, the power of this statistic tends to increase quickly relative to its size. The conclusions of Teverovsky, Taqqu, and Willinger (1999) should be qualified by the fact that these authors consider excessive values for q (e.g., 50).

to the rule that they use to forecast prices. One rule is based on economic fundamentals, and those who follow it are called "fundamentalists," and the other is based on extrapolation and is used by "chartists." The important feature of these models is that individuals change from being fundamentalists and become chartists and vice versa. Thus, the groups are not fixed in size, and this has consequences for market behavior.

It is important to note that the models here, although of a sequential nature, are still equilibrium models. In other words, at each point of time the level of exchange rate is such that the total volume demanded is equal to the total volume supplied. On a daily basis, this may not be unreasonable, but if we take price data at shorter intervals, this sort of model becomes much less realistic. As soon as we look at prices at very high frequencies, we are obliged to ask what those prices represent. This will depend on the type of market organization but will at best reflect the terms at which transactions are made and not at all short-term equilibrium. In an electronic trading system, if the best offer of a particular share is 500 at $100, it may be faced with a demand for 2,000 at that price. Nevertheless 500 of the demand will be served, and that price will be recorded, even though it cannot reasonably be considered an equilibrium price. Thus, to model prices of this sort using an equilibrium process, even though this is common practice, is unrealistic. In particular, the foreign-exchange market involves "market makers" trading independently and quoting "bids" and "asks." The published rates reflect some average of these.[8]

Why is this important? Simply because the sort of model here does not capture the statistical features of high-frequency data, and for the reasons just given, this is not surprising.

If the markets are efficient, the expected price $E(P_{t+1})$ of an asset at time $t+1$ conditional on the information set I_t is given by

$$E(P_{t+1} \mid I_t) = P_t \tag{4.1}$$

In our setting, agents do not consider markets to be efficient and then assume that they can predict the next price level, that is,[9]

$$E(P_{t+1} \mid I_t) = \Delta P_{t+1} \mid I_t + P_t \tag{4.2}$$

where $\Delta P_{t+1} \mid I_t$ is the predicted price change at time $t+1$, given the information set I_t. Let P_t be the exchange rate at time t, chartists make the assumption that the exchange rate in the next period is a convex linear function of the previous prices, that is,

$$E^c(P_{t+1} \mid I_t) = \sum_{j=0}^{M_c} h_j P_{t-j} \quad \text{with} \quad \sum_{j=0}^{M_c} h_j = 1 \tag{4.3}$$

where h_j, $j = 0, \ldots, M_c$, are constants and M_c is the memory of the chartists. On the other hand, fundamentalists forecast the next price as

$$E^f(P_{t+1} \mid I_t) = \bar{P}_t + \sum_{j=1}^{M_f} \nu_j (P_{t-j+1} - \bar{P}_{t-j}) \tag{4.4}$$

where ν_j, $j = 1, \ldots, M_f$, are positive constants, representing the degree of reversion to the fundamentals, M_f is the memory of the fundamentalists. This series of "fundamentals" can be thought of as the price if it were only to be explained by a set of relevant exogenous variables. We assume that the fundamentals $\bar{P}_t$ follow a random walk:

$$\bar{P}_t = \bar{P}_{t-1} + \varepsilon_t \quad \text{with} \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \tag{4.5}$$

[8]For details of the mechanics of foreign-exchange markets see, e.g., Grabbe (1992).
[9]The basic features of the model would not be changed if we allowed for some more sophisticated extrapolary processes.

Individuals i, $i = 1, \ldots, N$, have a utility function given by

$$U(W_{t+1}^i) = E(W_{t+1}^i) - \mu V(W_{t+1}^i) \tag{4.6}$$

where μ denotes the risk aversion coefficient and $E(.)$ and $V(.)$ denote the expectation and variance operators. Agents have the possibility of investing at home in a risk-free asset or investing abroad in a risky asset. This foreign investment involves two risks, that of an exchange rate change and that intrinsic in the foreign asset.

Denote by ρ_t the foreign interest rate, d_t^i the demand by the ith individual for foreign currency, and r the domestic interest rate. The exchange rate P_t and the foreign interest rate ρ_t are considered by agents to be independent random variables, with

$$\rho_t \sim N(\rho, \sigma_\rho^2) \quad \text{with} \quad \rho_t > r \tag{4.7}$$

Hence, the cumulated wealth of individual i at time $t+1$, W_{t+1}^i, is given by

$$W_{t+1}^i = (1 + \rho_{t+1}) P_{t+1} d_t^i + (W_t^i - P_t d_t^i)(1 + r) \tag{4.8}$$

Thus, we have

$$E(W_{t+1}^i \mid I_t) = (1 + \rho) E^i(P_{t+1} \mid I_t) d_t^i + (W_t^i - P_t d_t^i)(1 + r) \tag{4.9}$$

$$V(W_{t+1}^i \mid I_t) = (d_t^i)^2 \zeta_t \quad \text{where} \quad \zeta_t = V(P_{t+1}(1 + \rho_{t+1})) \tag{4.10}$$

Demand d_t^i is found by maximizing utility and writing the first-order condition

$$(1 + \rho) E^i(P_{t+1} \mid I_t) - (1 + r) P_t - 2\zeta_t \mu d_t^i = 0 \tag{4.11}$$

where $E^i(. \mid I_t)$ denotes the expectation of an agent of type i. Let k_t be the proportion of fundamentalists at time t; the market demand is then

$$d_t = \frac{(1 + \rho)(k_t E^f(P_{t+1} \mid I_t) + (1 - k_t) E^c(P_{t+1} \mid I_t)) - (1 + r) P_t}{2\zeta_t \mu} \tag{4.12}$$

Now consider the exogenous supply of foreign exchange X_t; the market is then in equilibrium if aggregate supply is equal to aggregate demand (i.e., $X_t = d_t$), which gives

$$(1 + r) P_t = (1 + \rho)(k_t E^f(P_{t+1} \mid I_t) + (1 - k_t) E^c(P_{t+1} \mid I_t)) - 2\zeta_t \mu X_t \tag{4.13}$$

We assume that $2\zeta_t \mu X_t / (1 + \rho) = \gamma \bar{P}_t$. If $M_f = M_c = 1$, then the equilibrium price is given by

$$P_t = \frac{k_t - \gamma}{A} \bar{P}_t - \frac{k_t \nu_1}{A} \bar{P}_{t-1} + \frac{(1 - k_t) h_1}{A} P_{t-1} \tag{4.14}$$

with

$$A = \frac{1 + r}{1 + \rho} - (1 - k_t) h_0 - k_t \nu_1 \tag{4.15}$$

Thus, for our so-called Havana–India model, the foreign-exchange rate P_t is a combination with varying coefficients of the previous price P_{t-1} and the fundamentals $\bar{P}_t$ and $\bar{P}_{t-1}$.

Since this paper has been motivated by the similarity of the long-memory component of the volatilities and covolatilities of FX rates, we wish to check whether our model is able to generate this empirical characteristic. We then consider a second exchange rate, denoted by P_t^*, which depends on a series of fundamentals $\bar{P}_t^*$, a foreign interest rate ρ_t^*, and two forecast functions with parameters ν_1^*, h_0^*, and h_1^*. We impose a common restriction on the two processes P_t and P_t^* by assuming that the proportion of fundamentalists k_t is the same for both processes. This assumption is reasonable, as being a fundamentalist means relying on fundamentals, whatever they are, provided that there is no good reason for not relying on a particular series of fundamentals.

Thus, the process P_t^* is defined as

$$P_t^* = \frac{k_t - \gamma^*}{A^*}\bar{P}_t^* - \frac{k_t \nu_1^*}{A^*}\bar{P}_{t-1}^* + \frac{(1-k_t)b_1^*}{A^*}P_{t-1}^* \tag{4.16}$$

where

$$A^* = \frac{1+r}{1+\rho^*} - (1-k_t)b_0^* - k_t\nu_1^* \tag{4.17}$$

The last building block of the model consists in introducing the process governing the evolution of k_t (i.e., the proportion of agents making a forecast based on fundamentals in the whole population). It is assumed that

1. Agents interact.
2. Agents communicate their beliefs on the next-period forecast through a particular epidemiologic process introduced by Föllmer.

Since the parameters of the epidemiologic model are independent of the previous parameters of the model, the proportion of fundamentalists and the forecasts of agents are independent of the economic variables.

Let N be the total number of agents and ϑ_t be the number of agents with a fundamentalist forecast at time t. We assume that pairs of agents meet at random and that the probability that the first agent is converted to the opinion of the second one is equal to $(1-\delta)$. Furthermore, each agent can independently change his opinion with probability ξ. This probability ensures that the process is not trapped in the extremes (i.e., all agents are chartists or all agents are fundamentalists).

Given that the state of the process is summarized by the value of ϑ_t, its evolution is defined by the following transition matrix:[10]

$$\Pr(\vartheta, \vartheta+1) = \left(1 - \frac{\vartheta}{N}\right)\left(\xi + (1-\delta)\frac{\vartheta}{N-1}\right) \tag{4.18}$$

$$\Pr(\vartheta, \vartheta-1) = \frac{\vartheta}{N}\left(\xi + (1-\delta)\frac{N-\vartheta}{N-1}\right) \tag{4.19}$$

$$\Pr(\vartheta, \vartheta) = 1 - \Pr(\vartheta, \vartheta+1) - \Pr(\vartheta, \vartheta-1) \tag{4.20}$$

After the meetings, the proportion of fundamentalists is equal to ϑ_t/N. Agents observe this proportion with error, however; that is, agent i observes $k_{i,t}$, with

$$k_{i,t} = \frac{\vartheta_t}{N} + \varepsilon_{i,t} \quad \text{with} \quad \varepsilon_{i,t} \sim N(0, \sigma_\vartheta^2) \tag{4.21}$$

If agent i observes that $k_{i,t} \geq 0.5$, then he will make a fundamentalist forecast; otherwise he will make a chartist forecast. Thus, the proportion k_t of agents making a fundamentalist forecast is given by

$$k_t = N^{-1}\sum_{i=1}^{N}\mathbf{1}_{(k_{i,t}\geq 0.5)} \tag{4.22}$$

where $\mathbf{1}_{(cond)}$ denotes the indicator function, which is equal to 1 if *cond* is true and 0 otherwise.

[10]The limit distribution of the Markov chain defined by this transition matrix is a beta distribution. This result, proved by Föllmer, is reproduced in Kirman (1991).

The herding behavior of the process k_t on the extremes depends on ξ and σ_ϑ. If ξ becomes smaller, then the process k_t will spend more time on the extremes 0 and 1. The parameter σ_ϑ measures the accuracy of observation of the proportion of fundamentalists (see Equation (4.21)). If σ_ϑ becomes smaller, the prevailing opinion is observed with more accuracy, which results in massive swings of opinion. There are two related reasons for such behavior. First, Keynes pointed out that individual trades are concerned about what "market opinion" is rather than about fundamental values. This, he argued, is because it is less risky to be wrong with the crowd rather than being wrong alone. Alternatively one could think of a Nash equilibrium in which if all people have the same opinion, no one has an incentive to deviate. As we will see in the next section, these two parameters govern the level of long-range dependence in the volatility of the simulated returns.

5 Monte Carlo Analysis of the Process

5.1 Characteristics of the simulations

We generate 10,000 samples of 3,000 observations, which is the typical size for daily financial time series. We adjust the parameters of the model so that the series of returns $R_t = \ln(P_t/P_{t-1})$ generated by the epidemiologic model is $I(0)$ and the volatility of the series of simulated returns displays long-range dependence, that is, is $I(d)$ with $d \in (0, 1/2)$. We choose as proxy of the volatility the series of absolute returns $|R_t|$ and squared returns R_t^2.

We detect the presence of long memory by using the tests for $I(0)$ against $I(d)$ alternatives presented in section 3. For the test based on the partial-sum process (i.e., Lo's test, the KPSS test, and the V/S test), we consider the truncation orders $q = 0, 1, 2, \ldots, 5, 10, 15, \ldots, 30$ for the HAC variance estimator. For the Lobato and Robinson (1998) test, we use a grid of bandwidths $m = 60, 84, 108, 132, 156$ as in Lobato and Savin 1998. We tune the parameters of the model so that the degree of long memory, estimated with Robinson's (1995) estimator and the optimal bandwidth m_{opt} by Henry and Robinson (1996), is the same as the one observed in financial time series. For this Gaussian estimator, we use the optimal bandwidth m_{opt} and the grid $m = 84, 108, 132, 156$.

We choose the following values for the parameters of the model:

- Number of agents N = 1,000.
- At time $t = 0$, the number of fundamentalists is equal to the number of chartists, (i.e., $k_0 = 0.5$).
- $P_0 = 1{,}000$, initial value of the exchange rate.
- $\bar{P}_0 = 1{,}050$, initial value of the fundamentals series.
- $\sigma_\varepsilon^2 = 10.0$.
- Annual foreign interest rate $\rho = 0.07$; the daily foreign interest rate is then equal to 0.00018538.
- Annual domestic interest rate $r = 0.04$; the daily domestic interest rate is then equal to 0.000133668.
- $\nu_1 = 0.59$, $b_0 = 0.625$, $b_1 = 1 - b_0$.
- δ = 0.010.
- ξ = 0.000325.
- σ_ϑ^2 = 0.33.

For the bivariate process (P_t, P_t^*), the parameters of the process P_t remain the same, and the parameters of the process P_t^* are

- ρ^* = 0.08%; at an annual rate, the daily rate is then equal to 0.00021087.
- $P_0^* = 1{,}500$, initial value of the second series of exchange rate.

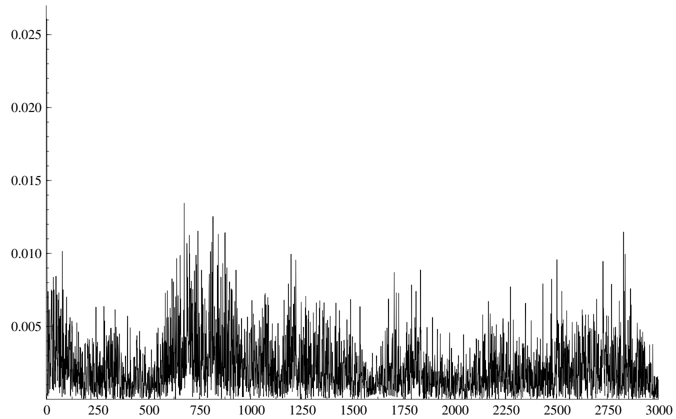

Figure 1
Series of simulated absolute returns $|R_t|$.

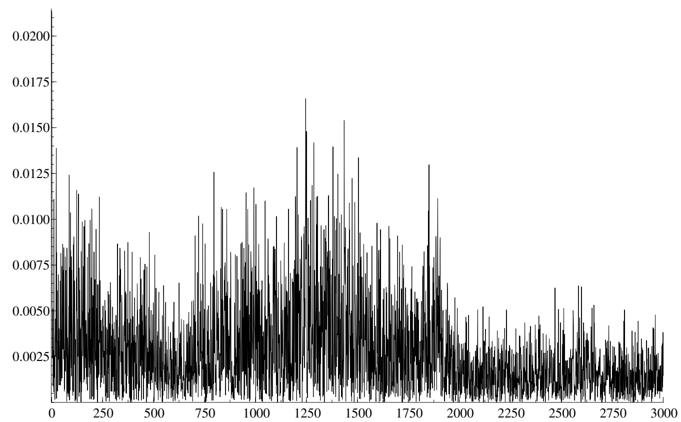

Figure 2
Series of simulated absolute returns $|R_t|$.

- $\bar{P}_0^* = 1{,}550$, initial value of the second series of fundamentals, $\sigma_\varepsilon^2 = 15.0$.
- $\nu_1^* = 0.58$, $h_0^* = 0.625$, $h_1^* = 1 - h_0^*$.

5.2 Simulation results

The series of simulated returns match the empirical properties of the two first moments of absolute returns. Unlike standard long-memory processes, the series of simulated absolute returns are not trended. Figures 1 and 2 display two series of simulated absolute returns, which are similar to what is empirically observed (see Figures 3 and 4). For Figures 1 and 2 the estimated values of d, obtained with Robinson's (1995) Gaussian estimator and the optimal bandwidth, are respectively $\hat{d} = 0.4425$ and $\hat{d} = 0.3914$. A standard long-memory process with such degrees of dependence will have a marked trend.

The series of simulated absolute returns, however, display strong dependence. Figure 5 displays the autocorrelation function (ACF) of a series of absolute simulated returns that resembles the empirical ACF of absolute returns displayed in Figures 6 and 7: there is a significant autocorrelation, as observed with real data. Simulation results reported in Tables 4 and 5 show that the four statistical tests accept 95% of the time the null hypothesis of no long-range dependence for the differenced series $(1 - L)P_t$.

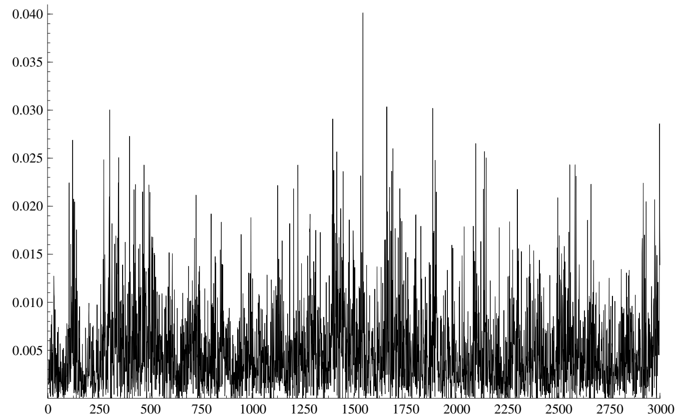

Figure 3
Series of absolute returns $|R_t|$ Deutsche mark–dollar.

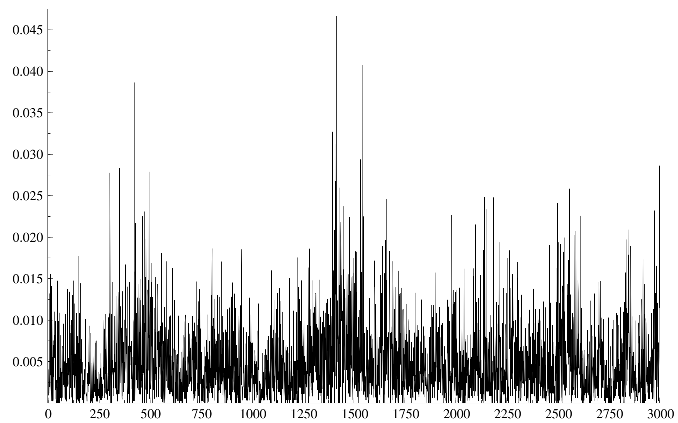

Figure 4
Series of absolute returns $|R_t|$ pound–dollar.

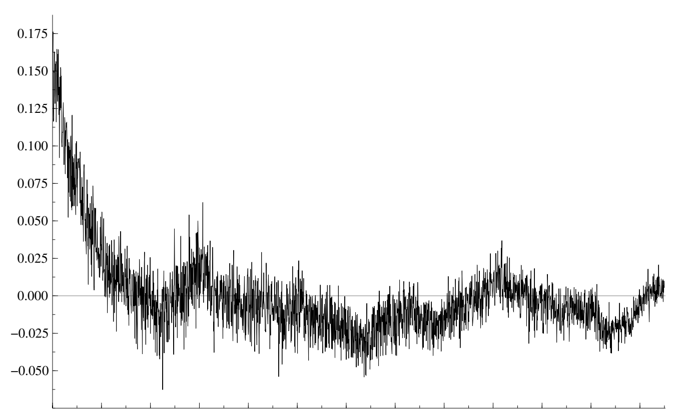

Figure 5
Autocorrelation function of the absolute returns $|R_t|$ of a simulation.

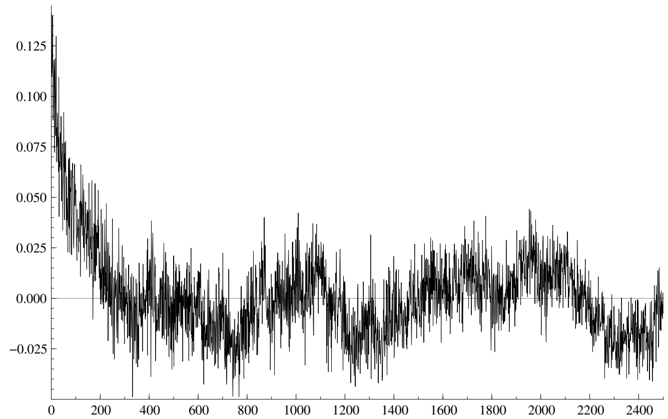

Figure 6
Autocorrelation function of the absolute returns $|R_t|$ on pound–dollar.

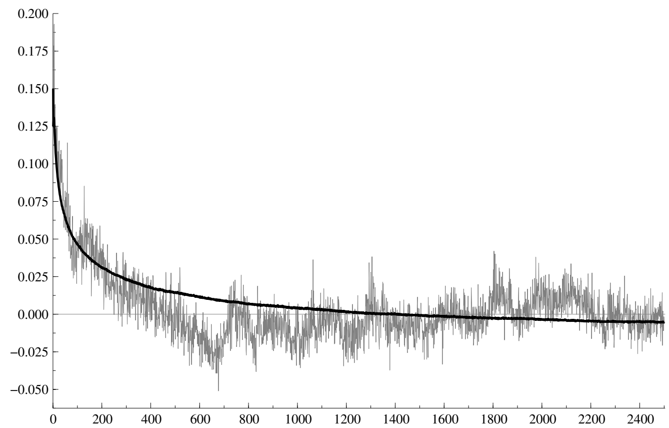

Figure 7
Comparison of ACF of $|R_t|$ (in grey) with the averaged ACF of 2,000 simulations of a FINGARCH-t process (in black).

Table 4
Lobato-Robinson test

m	R_t: $\Pr(d = 0)$	$\lvert R_t \rvert$: $\Pr(d > 0)$	R_t^2: $\Pr(d > 0)$
60	0.9534	0.9940	0.9936
84	0.9444	0.9947	0.9946
108	0.9317	0.9956	0.9960
132	0.9194	0.9959	0.9955
156	0.9051	0.9963	0.9959

Note: Test size 5%.

Table 6 displays the mean of the degree of long memory estimated with Robinson's (1995) Gaussian estimator. We observe that the Monte Carlo standard deviations of the Gaussian estimator are greater than the theoretical standard deviations $(2\sqrt{m})^{-1}$ obtained under some assumptions on the process generating the data (see Robinson 1995): for our sample size, the theoretical standard deviations for the bandwidths 84, 108, 132, and 156 are equal to 0.0545, 0.0481, 0.0435, and 0.0400, respectively. This means that our microeconomic model–based data-generating process (DGP) is more complex than a standard linear long-memory process.

Table 5
Tests for long-range dependence on the series of differenced prices, absolute simulated returns $|R_t|$, and squared simulated returns R_t^2

	R_t: $\Pr(d=0)$			$\lvert R_t\rvert$: $\Pr(d>0)$			R_t^2: $\Pr(d>0)$		
q	R/S	KPSS	V/S	R/S	KPSS	V/S	R/S	KPSS	V/S
0	0.9268	0.9494	0.9403	0.9955	0.9891	0.9937	0.9960	0.9886	0.9936
1	0.9336	0.9491	0.9442	0.9944	0.9865	0.9927	0.9946	0.9861	0.9922
2	0.9416	0.9482	0.9470	0.9934	0.9850	0.9917	0.9937	0.9847	0.9912
3	0.9470	0.9484	0.9495	0.9928	0.9844	0.9912	0.9931	0.9839	0.9908
4	0.9506	0.9485	0.9506	0.9926	0.9834	0.9903	0.9926	0.9829	0.9902
5	0.9530	0.9495	0.9530	0.9925	0.9825	0.9901	0.9918	0.9819	0.9899
10	0.9610	0.9501	0.9575	0.9913	0.9790	0.9893	0.9896	0.9778	0.9877
15	0.9618	0.9487	0.9595	0.9891	0.9752	0.9865	0.9860	0.9719	0.9855
20	0.9643	0.9491	0.9595	0.9850	0.9681	0.9837	0.9834	0.9663	0.9827
25	0.9657	0.9499	0.9591	0.9810	0.9617	0.9801	0.9784	0.9590	0.9790
30	0.9655	0.9499	0.9602	0.9767	0.9537	0.9762	0.9723	0.9508	0.9752

Note: Test size 5%.

Table 6
Gaussian estimates of d

m	R_t	$\lvert R_t\rvert$	R_t^2
m_{opt}	0.0020 (0.0578)	0.3509 (0.0976)	0.3219 (0.0944)
84	−0.0063 (0.0712)	0.4806 (0.1205)	0.4483 (0.1186)
108	−0.0032 (0.0629)	0.4350 (0.1100)	0.4049 (0.1078)
132	−0.0005 (0.0580)	0.4009 (0.1021)	0.3723 (0.0996)
156	0.0013 (0.0550)	0.3742 (0.0962)	0.3468 (0.0931)

Note: Monte Carlo S.E. in parentheses.

The use of a stochastic process for modeling the evolution of the process k_t allows us to control the link between herding behavior and long memory. This makes the analysis easier than in Kirman (1999) and Gaunersdorfer and Hommes (2000), which used an evolutionary mechanism based on the relative performance of the forecast functions of chartists and fundamentalists. We consider several values for the parameter ξ, which is the probability that an agent independently changes his opinion, and the parameter σ_ϑ^2, which represents the accuracy of observation of the proportion of fundamentalists (see Equation (4.21)). When $\sigma_\vartheta^2 = 0.33$ and $\xi = 0.000325$, the estimated degree of long memory d in the absolute returns is equal to 0.35, whereas when $\sigma_\vartheta^2 = 0.60$ and $\xi = 0.000325$, the estimated value of d decreases to 0.30, and when $\xi = 0.01$ and $\sigma_\vartheta^2 = 0.33$, this degree decreases to 0.20. Thus, the estimated degree of long-range dependence of the model is linked to the herding behavior of agents.

Table 7 reports the estimated values of d from the three "pox-plot"-based semiparametric estimators, which do not differ too much from the results obtained from the local Whittle estimator.[11] We can conclude that our DGP differs from the long-memory linear ARCH model by Giraitis, Robinson and Surgailis (2000), which is not surprising.

Table 7
"Pox-plot" estimates of d based on the squared returns series R_t^2

	R/S estimate of d	V/S estimate of d	KPSS estimate of d
d	0.3086 (0.0872)	0.3259 (0.1043)	0.3870 (0.1172)

Note: Monte Carlo S.E. in parentheses.

[11] Interestingly, the Monte Carlo standard errors differ slightly from the ones of the long-memory linear ARCH process (See Giraitis et al. 2000).

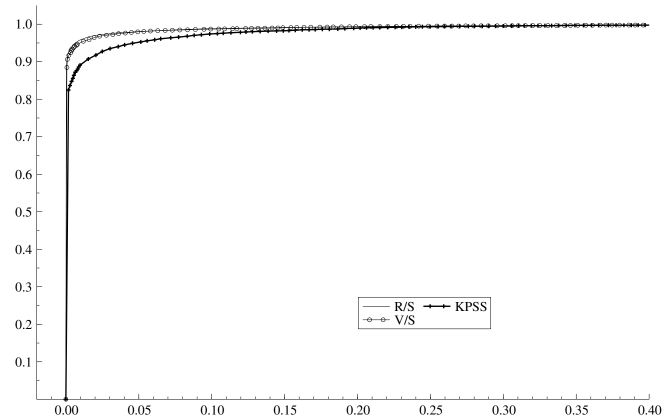

Figure 8
Power-size curves: Absolute returns $|R_t|$.

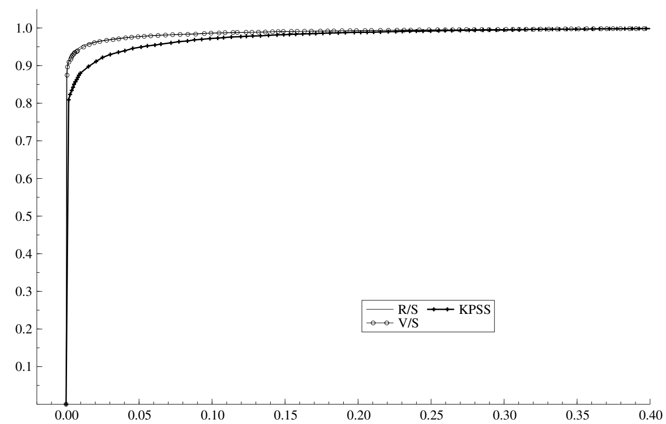

Figure 9
Power-size curves: Squared returns R_t^2.

Results in Tables 4 and 5 show that the Lobato-Robinson test, Lo's test, the KPSS test, and the V/S test detect the presence of long memory in the simulated series of absolute and squared returns. The Lobato–Robinson test, the V/S test and Lo's test have the same power and have more power than the KPSS test. Giraitis et al. (forthcoming) conjectured that this higher power is due to the smaller variance of the V/S statistic.

When interpreting these results, we have to keep in mind that the performance of these asymptotic tests was originally assessed with respect to a standard DGP. Given that we apply them to a nonstandard DGP, we have to adjust them on the correct size basis. Davidson and MacKinnon (1998) suggest drawing the size-power trade-off curve, which graphs the probability of rejecting the null hypothesis when it is false against the probability of rejecting the null hypothesis when it is true. Figures 8 and 9 display the size-power curves that are obtained by plotting the empirical distribution function (EDF), $\tilde{F}(x)$, of the P-values of the DGP, which does not satisfy the null hypothesis, against the EDF, $\hat{F}(x)$, of the P-values of a DGP that satisfies this null hypothesis. If the test is pivotal (i.e., its distribution is independent of any unknown feature of the process generating the data), the choice of the DGP satisfying the null hypothesis is not important. If the test is not pivotal, however, Davidson and MacKinnon (1996) suggest choosing the "pseudo-null" DGP, which is, in the set of DGPs satisfying the null hypothesis, the closest, according to the Kullback-Leibler criterion, to the DGP that does not satisfy the null hypothesis.

Table 8
Gaussian estimates of d for the bivariate series of simulated absolute returns $|R_t|$, $|R_t^*|$, $\sqrt{|R_t R_t^*|}$

m	$\|R_t\|$	$\|R_t^*\|$	$\sqrt{\|R_t R_t^*\|}$
m_{opt}	0.3509 (0.0976)	0.3457 (0.0987)	0.3939 (0.1016)
84	0.4806 (0.1205)	0.4748 (0.1203)	0.5297 (0.1240)
108	0.4350 (0.1100)	0.4291 (0.1103)	0.4812 (0.1138)
132	0.4009 (0.1021)	0.3947 (0.1031)	0.4445 (0.1061)
156	0.3742 (0.0962)	0.3687 (0.0973)	0.4163 (0.1004)

Note: Test size 5%.

For $q = 0$, the size-power curves for all the statistics are indistinguishable; thus we display the curves for $q = 30$. The size-power curves for Lo's statistic and the V/S test are over the curve of the KPSS statistic. For small samples (e.g., 500 observations), the size-power curve of the V/S test is over the two other curves (see Kirman and Teyssière forthcoming). The V/S statistic is slightly less sensitive to the choice of q than Lo's statistic.

Finally, Table 8 gives the simulation results for the bivariate process (R_t, R_t^*), with $R_t = \ln(P_t/P_{t-1})$ and $R_t^* = \ln(P_t^*/P_{t-1}^*)$. These results show that the estimated values of d are close for the series of volatilities $|R_t|$, $|R_t^*|$ and covolatilities $\sqrt{|R_t, R_t^*|}$ for all the values of the bandwidth parameter m. Our model is then able to generate the empirical property of common long-range dependence in the volatilities and covolatilities of FX rates.

Figures 10 and 11 display two examples of the evolution of the process k_t. The process never herds on the extremes 0 and 1, which is consistent with what is empirically observed on financial markets: agents never all swing together to one belief. There is a tendency for one view to predominate at any point in time, but there is always a minority with the other view, and in time this minority takes over.

6 Conclusion

We have presented here a new class of models of financial markets based on the idea that individual agents interact stochastically. These models replicate the empirically observed characteristics of daily exchange rate series. These characteristics are that the returns are uncorrelated, but the absolute returns and squared returns display long memory.

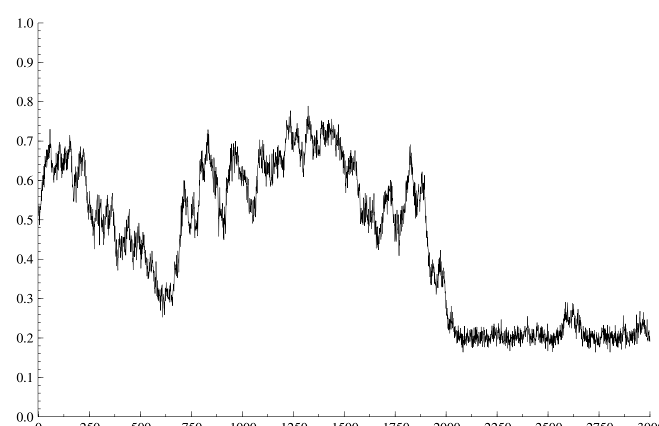

Figure 10
Evolution of the process k_t.

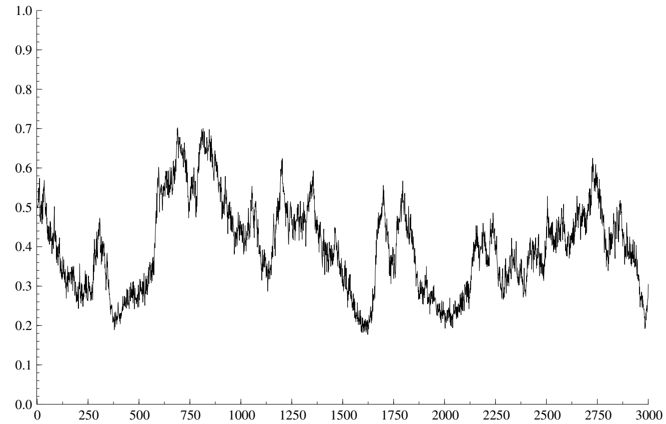

Figure 11
Evolution of the process k_t.

We have explained the nature of long memory and shown how it may be detected in the sort of nonstandard data-generating process yielded by our models. The fact that the process is nonstandard means that we had to use nonparametric and semiparametric methods. Although Evans (1991) pointed out that some bubble-like features that were deliberately introduced into time series would not be detected by standard procedures, he did not provide an economic model that would generate this structure.

Our models generate "herding behavior" and swings of opinion that give rise to bubble-like features (see Kirman and Teyssière 2001, forthcoming). The important feature of these models, however, from the point of view of this paper, is that they also generate the sort of long memory that can be detected and is actually present in empirical series.

What is most interesting is that this sort of long memory seems to be intimately linked to the tendency of the markets' participants to herd on the extremes. Thus bubbles seem to be linked to long memory through this herding behavior.

References

Aoki, M. (1996). *A New Approach to Macroeconomic Modeling.* New York: Cambridge University Press.

Avery, C., and P. Zemsky (1998a). "Multidimensional uncertainty and herd behavior in financial markets." *American Economic Review*, 88: 724–748.

Avery, C., and P. Zemsky (1998b). "Herding and manipulation in a financial market." Preprint.

Baillie, R. T., T. Bollerslev, and H. O. Mikkelsen (1996). "Fractionally integrated generalized autoregressive conditional heteroskedasticity." *Journal of Econometrics*, 74: 3–30.

Banerjee, A. (1992). "A simple model of herd behavior." *Quarterly Journal of Economics*, 107: 797–818.

Beran, J. (1994). *Statistics for Long-Memory Processes.* London: Chapman and Hall.

Bikhchandani, S., D. Hirshleifer, and I. Welch (1992). "A theory of fads, fashion, custom and cultural changes as informational cascades." *Journal of Political Economy*, 100: 992–1027.

Blanchard, O., and M. W. Watson (1982). "Bubbles, rational expectations and financial markets." in P. Wachtel (ed.), *Crisis in the Economic and Financial System.* Lexington, Mass: Lexington Books, 295–315.

Box-Steffenmaier, J. M., and R. M. Smith (1996). "The dynamics of aggregate partisanship." *American Political Science Review*, 90: 567–580.

Breidt, F. J., N. Crato, and P. de Lima (1998). "On the detection and estimation of long-memory in stochastic volatility." *Journal of Econometrics*, 83: 324–352.

Byers, D., J. Davidson, and D. Peel (1997). "Modelling political popularity: An analysis of long range dependence in opinion poll series." *Journal of the Royal Statistical Society*, A 160: 471–490.

Dacorogna, M. M., U. A. Müller, R. J. Nagler, R. B. Olsen, and O. V. Pictet (1993). "A geographical model for the daily and weekly seasonal volatility in the foreign exchange market." *Journal of International Money and Finance*, 12: 413–438.

Dacorogna, M. M., U. A. Müller, O. V. Pictet, and C. G. De Vries (1995). "The distribution of extremal foreign exchange rate returns in extremely large data sets." Research paper UAM 1992-10-22. Zurich: Olsen & Associates.

Davidson, R., and J. G. MacKinnon (1996). "The power of bootstrap tests." Discussion paper no. 937. Kingston, Ontario: Queens Institute for Economic Research.

Davidson, R., and J. G. MacKinnon (1998). "Graphical methods for investigating the size and power of hypothesis tests." *Manchester School*, 66: 1–26.

Day, R., and B. Huang (1990). "Bulls, bears and market sheep." *Journal of Economic Behavior and Organization*, 14: 299–329.

Dickey, D. A., and W. A. Fuller (1979). "Distribution of the estimators for autoregressive time series with a unit root." *Journal of the American Statistical Association*, 74: 427–431.

Ding, Z., and C. W. J. Granger (1996). "Modelling volatility persistence of speculative returns: A new approach." *Journal of Econometrics*, 73: 185–215.

Durlauf, S. N. (1997). "Statistical mechanics approaches to socioeconomic behaviour." In W. B. Arthur, S. N. Durlauf, and D. A. Lane (eds.), *The Economy as an Evolving Complex System*. Reading, Mass.: Addison Wesley, 81–104.

Evans, G. W. (1991). "Pitfalls in testing for explosive bubbles in asset prices." *American Economic Review*, 81: 922–930.

Gallant, A. R., C.-T. Hsu, and G. Tauchen (1998). "Calibrating volatility diffusions and extracting integrated volatility." In *Proceedings of the Second High Frequency Data in Finance Conference, HFDF-II*, Olsen & Associates, 1–3 April 1998, Zurich, Switzerland.

Gaunersdorfer, A., and C. H. Hommes (2000). "A nonlinear structural model for volatility clustering." Working paper no. 00-02. Amsterdam: Centre for Non-linear Dynamics in Economics and Finance.

Giraitis, L., and D. Surgailis (2001). "ARCH-type bilinear models with double long-memory." Preprint.

Giraitis, L., P. S. Kokoszka, and R. Leipus (2000). "Stationary ARCH models: Dependence structure and central limit theorem." *Econometric Theory*, 16: 3–22.

Giraitis, L., P. S. Kokoszka, R. Leipus, and G. Teyssière (2000). "Semiparametric estimation of the intensity of long-memory in conditional heteroskedasticity." *Statistical Inference for Stochastic Processes*, 3: 113–128.

Giraitis, L., P. S. Kokoszka, R. Leipus, and G. Teyssière (forthcoming). "Rescaled variance and related tests for long-memory in volatility and levels." *Journal of Econometrics*.

Giraitis, L., P. M. Robinson, and D. Surgailis (2000). "A model for long-memory conditional heteroskedasticity." *Annals of Applied Probability*, 10: 1002–1024.

Grabbe, J. O. (1992). *International Financial Markets*. New York: Prentice Hall.

Granger, C. W. J. (1980). "Long-memory relationships and the aggregation of dynamic models." *Journal of Econometrics*, 14: 227–238.

Granger C. W. J., and Z. Ding (1995). "Some properties of absolute returns, an alternative measure of risk." *Annales d'Économie et de Statistique*, 40: 67–91.

Granger, C. W. J., and Z. Ding (1996). "Varieties of long-memory models." *Journal of Econometrics*, 73: 61–77.

Granger, C. W. J., and R. Joyeux (1980). "An introduction to long-memory time series models and fractional differencing." *Journal of Time Series Analysis*, 1: 15–29.

Hamilton, J. D. (1989). "A new approach to the economic analysis of nonstationary time series and the business cycle." *Econometrica*, 57: 357–384.

Harvey, A. C. (1998). "Long-memory in stochastic volatility." In J. Knight and S. Satchell (eds.), *Forecasting Volatility in Financial Markets*. Oxford: Butterworth-Heineman, 307–320.

Henry, M. (2001a). "Robust automatic bandwidth for long-memory." *Journal of Time Series Analysis*, 22: 293–316.

Henry, M. (2001b). "Averaged periodogram spectral estimation with long-memory conditional heteroskedasticity." *Journal of Time Series Analysis*, 22: 431–459.

Henry, M., and R. Payne (1997). "An investigation of the long range dependence in intra-day foreign exchange rate volatility." Discussion paper no. 264. London: London School of Economics Financial Market Group.

Henry, M., and P. M. Robinson (1996). "Bandwidth choice in Gaussian semiparametric estimation of long range dependence." In P. M. Robinson and M. Rosenblatt (eds.), *Athens Conference on Applied Probability and Time Series Analysis, In Memory of E. J. Hannan*, 2, LNS 115. New York: Springer Verlag, 220–232.

Hosking, J. R. M. (1981). "Fractional differencing." *Biometrika*, 68: 165–176.

Hurst, H. E. (1951). "Long-term storage capacity of reservoirs." *Transactions of the American Society of Civil Engineers*, 116: 770–799.

Kazakevičius, V., and R. Leipus (2002). "On stationarity in the ARCH(∞) model." *Econometric Theory*, 18: 1–16.

Kazakevičius, V., R. Leipus, and M.-C. Viano (2000). "Stability of random coefficients autoregressive conditionally heteroskedastic models." Preprint.

Kirman, A. (1991). "Epidemics of opinion and speculative bubbles in financial markets." In M. Taylor (ed.), *Money and Financial Markets*. Oxford: Blackwell, 354–368.

Kirman, A. (1993). "Ants, rationality and recruitment." *Quarterly Journal of Economics*, 108: 137–156.

Kirman, A. (1999). "On the transitory nature of gurus." Preprint.

Kirman, A., and G. Teyssière (2001). "Testing for bubbles and change–point." Preprint.

Kirman, A., and G. Teyssière (forthcoming). "Bubbles and long range dependence in asset prices volatilities," in C. H. Hommes, R. Ramer, and C. Withagen (eds.), *Equilibrium, Markets and Dynamics*. Springer Verlag.

Künsch, H. R. (1987). "Statistical aspects of self-similar processes." In Y. Prohorov and V. V. Sazanov (eds.), *Proceedings of the First World Congress of the Bernoulli Society*, vol. 1. Utrecht: VNU Science Press, 67–74.

Kwiatkowski, D., P. C. B. Phillips, P. Schmidt, and Y. Shin (1992). "Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic series have a unit root?" *Journal of Econometrics*, 54: 159–178.

Lee, D., and P. Schmidt (1996). "On the power of the KPSS test of stationarity against fractionally-integrated alternatives." *Journal of Econometrics*, 73: 285–302.

Lo, A. W. (1991). "Long-term memory in stock market prices." *Econometrica*, 59: 1279–1313.

Lobato, I., and P. M. Robinson (1998). "A nonparametric test for I(0)." *Review of Economic Studies*, 65: 475–495.

Lobato, I., and N. E. Savin (1998). "Real and spurious long-memory properties of stock market data." *Journal of Business and Economic Statistics*, 16: 261–283.

MacKinnon, J. G. (1994). "Approximate asymptotic distribution functions for unit-root and cointegration tests." *Journal of Business and Economic Statistics*, 12: 167–176.

MacKinnon, J. G. (1996). "Numerical distribution functions for unit-root and cointegration tests." *Journal of Applied Econometrics*, 11: 601–618.

Mandelbrot, B. (1963). "The variation of certain speculative prices." *Journal of Business*, 36: 384–419.

Mandelbrot, B. B., A. Fisher, and L. Calvet (1997). "A multifractal model of asset returns." Discussion paper no. 1164. New Haven: Cowles Foundation, Yale University.

Mandelbrot, B. B., and M. S. Taqqu (1979). "Robust R/S analysis of long-run serial correlation." In *42nd Session of the International Statistical Institute, Manila*, Book 2, 69–99.

Mandelbrot, B. B., and J. M. Wallis (1969). "Computer experiments with fractional Gaussian noises." *Water Resources Research*, 5: 228–267.

Newey, W. K., and K. D. West (1987). "A simple positive definite, heteroskedasticity and autocorrelation consistent covariance matrix." *Econometrica*, 55: 703–708.

Odaki, M. (1993). "On the invertibility of fractionally differenced ARIMA processes." *Biometrika*, 80: 703–709.

Ray, B. K., and R. S. Tsay (2000). "Common long-range dependence in daily stock volatilities." *Journal of Business and Economics Statistics*, 18: 254–262.

Robinson, P. M. (1991). "Testing for strong serial correlation and dynamic conditional heteroskedasticity in multiple regression." *Journal of Econometrics*, 47: 67–84.

Robinson, P. M. (1994a). "Time series with strong dependence." in C. A. Sims (ed.), *Advances in Econometrics, Sixth World Congress*. Cambridge: Cambridge University Press, 47–95.

Robinson, P. M. (1994b). "Efficient tests of nonstationary hypotheses." *Journal of the American Statistical Association*, 89: 1420–1437.

Robinson, P. M. (1994c). "Semiparametric analysis of long-memory time series." *Annals of Statistics*, 22: 515–539.

Robinson, P. M. (1995). "Gaussian semiparametric estimation of long range dependence." *Annals of Statistics*, 23: 1630–1661.

Robinson, P. M. (2001). "The memory of stochastic volatility models." *Journal of Econometrics*, 101: 195–218.

Robinson, P. M., and M. Henry (1999). "Long and short memory conditional heteroskedasticity in estimating the memory parameter in levels." *Econometric Theory*, 15: 299–336.

Sowell, F. (1990). "The fractional unit root distribution." *Econometrica*, 58: 495–505.

Taylor, S. J. (1986). *Modelling Financial Time Series*. New York: Wiley.

Teverovsky, V., M. S. Taqqu, and W. Willinger (1999). "A critical look at Lo's modified R/S statistic." *Journal of Statistical Planning and Inference*, 80: 211–227.

Teyssière, G. (1997). "Modelling exchange rates volatility with multivariate long-memory ARCH processes." Document du travail no. 97B03. Marseilles, France: GREQAM.

Teyssière, G. (1998a). "Multivariate long-memory ARCH modelling for high frequency foreign exchange rates." In *Proceedings of the HFDF-II Conference*, Olsen and Associates, 1–3 April 1998, Zurich, Switzerland.

Teyssière, G. (1998b). "Nonlinear and semiparametric long-memory ARCH." Preprint.

Welch, I. (1992). "Sequential sales, learning, and cascades." *Journal of Finance*, 47: 695–732.

Willinger, W., V. Paxson, and M. S. Taqqu (1998). "Self similarity and heavy tails: Structural modeling of network traffic." In R. J. Adler, R. E. Feldman, and M. S. Taqqu (eds.), *A Practical Guide to Heavy Tails: Statistical Techniques and Applications*. Boston: Birkhauser, 27–53.

Woodford, M. (1990). "Learning to believe in sunspots." *Econometrica*, 58: 277–307.