Long Memory in Foreign Exchange Rates Revisited

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ABSTRACT: There has been recent evidence for long memory in the changes of foreign exchange spot rates that is captured by the fractionally integrated ARMA model. This paper extends these investigations in several directions. First, the estimation procedure allows for GARCH errors. Second, in addition to the total period from 1973 to 1990 three subperiods are analyzed. Third, for the US-Dollar spot rates of the Deutsche Mark and the Swiss Franc ARFIMA model selection and estimation results for various observation frequencies are compared to ARFIMA specifications and their parameter values that are obtained from temporal aggregation. As a result the evidence for weak long memory in the changes of US-Dollar exchange rates is confirmed. However, long memory appears to be a property attached to the US currency since the analysis of the Deutsche Mark/Swiss Franc spot rate changes does not reveal any long memory.

KEYWORDS: time series analysis, long memory, ARFIMA models, GARCH models, foreign exchange rates, prediction, temporal aggregation

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1 INTRODUCTION

In the internationally linked economies of today, the behavior of foreign exchange rates is of crucial importance to international investors as it determines their real and financial profits as well as their investment opportunities. Any investment decision requires a rational investor to predict foreign exchange rates and the implied risk. While, during the last decade, there has been considerable progress in statistically modelling the latter, predicting flexible foreign exchange rates remains to be rather unsatisfactory if one aims at improving on the martingale forecasts that are given by the current rate, possibly combined with a drift.

Successfully applying more advanced statistical models for prediction would not only serve investors, but also economists in general as they would gain more insights into the empirical behavior of foreign exchange rates. In modern finance the pricing of derivatives requires assumptions on the price processes of the underlying assets. In practice, this price process should be statistically estimated. Thus, improving on the statistical description of the exchange rate process would immediately translate into a more reliable pricing of derivatives or an increase in the quality of optimal hedging policies against exchange rate risk. Moreover, a better knowledge of the empirical properties of exchange rates is an indispensable prerequisite for deriving a superior theoretical understanding of dynamic exchange rate behavior. This paper aims at providing more insights into the stochastic process underlying selected exchange rates.

There is strong evidence that foreign exchange rates exhibit strong nonlinearities. In particular, they are found to be conditionally heteroskedastic and unconditionally leptocurtic. However, Diebold and Nason (1990) are not able to exploit these nonlinearities for enhanced point prediction when using nonparametric techniques. Of course, within the class of linear models the martingale must be the best model for prediction if there is no autocorrelation in the data. Indeed, the hypothesis of uncorrelated increments cannot be rejected in many empirical investigations. For instance, using the heteroskedasticity-adjusted Box-Pierce Q test, Liu and He (1991) obtain this result for weekly US-Dollar spot rates of the Canadian Dollar, the French Franc, the Deutsche Mark, the Japanese Yen, and the British Pound for the period from August 7, 1974 to March 29, 1989. Employing the Box-Pierce statistic, Gaab (1983) finds similar results for some time series of daily exchange rate changes. He analyzes Deutsche Mark spot rates of the US Dollar, the British Pound, the Dutch Guilders, the French Franc for the period from January 2, 1974 until February 13, 1979 and various subperiods. Earlier studies finding no autocorrelation in exchange rate data include for example Cornell and Dietrich (1978) or Logue, Sweeney, and Willett (1978).

However, if the data show small autocorrelations of the same sign for many lags the acceptance of uncorrelated increments may be due to the behavior of the (heteroskedasticity-adjusted) Box-Pierce statistic. It is pointed out in Liu and He (1991) that in such a case the heteroskedasticity-consistent variance-ratio test developed by Lo and MacKinlay (1988) is more likely to reject the null of no serial autocorrelation. In fact, Liu and He (1991) find evidence for correlated weekly increments in the dollar spot rates of the Deutsche Mark and the Japanese Yen for the same period as above when they apply the variance-ratio test of Lo and MacKinlay.

Their results indicate that changes of foreign exchange rates may well exhibit autocorrelation, however, small and possibly not of the simple ARMA kind. One reason for the failure of the ARMA model could be that this class of linear time series models is only able to capture short memory processes. Consequently, ARMA models are unable to model stochastic dependence between distant observations. Processes of the latter kind are said to exhibit long memory and are defined by a non-absolutely summable autocovariance function (McLeod and Hipel (1978)). Indeed, using the R/S analysis, Booth, Kaen, and Koveos (1982) present evidence for long memory in daily changes of the US-Dollar spot rates of the British Pound, the French Franc, and the Deutsche Mark for the period from July 1, 1973 until June 30, 1979. Using the modified R/S test suggested by Lo (1991) that is robust to short-range dependence, Cheung (1993) confirms their findings for weekly exchange rate changes for a longer period that also includes the 80's.

Exploiting long memory for prediction, however, requires some kind of long memory model. As an attractive parameterization Granger and Joyeux (1980) and Hosking (1981) independently proposed the fractionally integrated ARMA(p, d, q) (also called ARFIMA(p, d, q)) model that is a direct generalization of the well known ARIMA(p, d, q) model by allowing the differencing parameter d to take real values instead of being restricted to the integer domain. Cheung (1993) was the first one who applied the ARFIMA model to foreign exchange rates. In his analysis of the weekly changes of US-Dollar spot rates of the British Pound, the Deutsche Mark, the Swiss Franc, the French Franc, and the Japanese Yen for the period from January 1974 to December 1989 he finds statistical evidence for long memory using various estimation techniques. However, the selected ARFIMA specifications do not outperform the random walk model in out-of-sample forecasts for the forecasting period 1990 and 1991.

This contradiction may indicate that Cheung's (1993) evidence of long memory in foreign exchange rates may be weaker than he claims. There exist at least two reasons why the finding of long memory in foreign exchange rates could be spurious. First of all, Cheung's (1993) estimation results may be biased since the estimation method chosen is not able to take into account conditional heteroskedasticity which is a well known feature of foreign exchange rates. Secondly, the finding of long memory could be caused by a specific but short episode in the time series of foreign exchange rates. Moreover, the generality of Cheung's results is restricted to US-Dollar spot rates and may be simply caused by the behavior of the US currency.

By re-examining the evidence for long memory in selected foreign exchange rates, this paper addresses all these issues. In order to deal with conditional heteroskedasticity in highfrequent data, an alternative approximate time domain maximum likelihood estimation method is applied that allows simultaneous estimation of an ARFIMA(p, d, q)-GARCH(P, Q) model. This method was suggested by Baillie, Chung, and Tieslau (1992). Secondly, the analysis of three subperiods in addition to the total period from January 31, 1973 to April 30, 1990 aims at detecting structural change. Moreover, results on the temporal aggregation of ARFIMA processes will be used as an additional check of the validity of the long memory hypothesis by investigating daily, weekly, monthly, and quarterly changes of exchange rates. Finally, in addition to the US-Dollar spot rates of the Deutsche Mark and the Swiss Franc, the pure European spot rate of the Swiss Franc/Deutsche Mark is analyzed.

In these investigations the strongest evidence for weak long memory in the changes of foreign exchange rates is found for the DM/US-Dollar rate followed by the evidence for the SF/US-Dollar spot rate. In contrast, the DM/SF spot rate does not exhibit any long memory within the class of ARFIMA models.

The paper is organized as follows. In section 2 the theory of the ARFIMA(p, d, q) model is introduced. The presentation includes results on temporal aggregation and an extension to GARCH errors. The empirical results are discussed in section 3. Details of the estimation and prediction techniques are summarized in an appendix. Section 4 concludes.

2 ARFIMA(p, d, q) MODELS

The ARFIMA(p, d, q) process $\{x_t\}$ satisfies the difference equation

$$\alpha(B)\nabla^d x_t = \beta(B)\varepsilon_t \tag{1}$$

with the disturbance process

$$\{\varepsilon_t\} \sim WN(0, \sigma^2) \tag{2}$$

being white noise with variance σ^2. Using the backshift operator B, the fractional differencing operator ∇^d can be written as an infinite AR polynomial

$$\nabla^d = \sum_{k=0}^{\infty} \pi_k B^k \tag{3}$$

with the AR coefficients π_k given by $\pi_0 = 1$ and

$$\pi_k = (-1)^k \frac{d(d-1)\dots(d-k+1)}{k!}, \quad k = 1, 2, \dots . \tag{4}$$

$\alpha(B) = 1 - \alpha_1 B - \alpha_2 B^2 - \dots - \alpha_p B^p$ and $\beta(B) = 1 + \beta_1 B + \beta_2 B^2 + \dots + \beta_q B^q$ denote the autoregressive and moving average polynomials, respectively. Both polynomials are assumed to have no common roots, and the roots of the AR polynomials $\alpha(z)$ and of the MA polynomial $\beta(z), z \in C$, are assumed to lie outside the unit circle. For $d \in (-0.5, 0.5)$ this process is invertible and causal (see Granger and Joyeux (1980), Hosking (1981) and Brockwell and Davis (1991, Def. 13.2.2, p. 524, Theorem 13.2.2, p. 525)). To obtain non-stationary processes with $d \geq 0.5, \nabla^d$ can be obtained by the combination of fractional differencing following equation (3) and integer differencing.

An explicit formula for computing the autocovariance function of an ARFIMA(p, d, q) process was derived by Sowell (1992). This procedure involves the calculation of the hypergeometric function if the AR part is non-zero.

For non-zero d, the autocovariance function $\gamma(\tau)$ declines hyperbolically

$$\gamma(\tau) \sim C\tau^{2d-1} \quad \text{as } \tau \to \infty, \tag{5}$$

where $C > 0$. (See Brockwell and Davis (1991, Theorem 13.2.2, pp. 525 - 6) for proofs.) In contrast, the traditional ARMA model corresponding to d equal to zero shows an autocovariance function that declines much faster at an exponential rate. As for positive d the autocovariance function (5) can be shown not to be absolutely summable (Brockwell and Davis (1991)), the ARFIMA(p, d, q) process exhibits long memory for $d > 0$ while $d = 0$ corresponds to the presence of short memory. The case of negative d is sometimes referred to as intermediate memory (Brockwell and Davis (1991)). Thus, the dependence between distant observations is solely determined by the memory parameter d.

In the frequency domain this memory classification corresponds to an infinite, finite, or zero spectral density at the origin for d greater than, equal to or smaller than zero, respectively (Brockwell and Davis (1991)). This can be directly seen from the spectral density function

$$f(\omega) = \frac{\sigma^2}{2\pi} \frac{|F_\beta(\omega)|^2}{|F_\alpha(\omega)|^2} |1 - e^{-i\omega}|^{-2d} \tag{6}$$

with F_α and F_β denoting the Fourier transforms of the AR and MA polynomial as it can be approximated for $\omega \to 0$ by

$$\lim_{\omega \to 0} f(\omega) = \begin{cases} \infty & \text{if} \quad d > 0. \\ 0 & \text{if} \quad d < 0. \end{cases} \tag{7}$$

Thus, by allowing to model short and long memory simultaneously, ARFIMA(p, d, q) processes suggest themselves as an ideal tool for detecting long memory.

If one observes the ARFIMA process $\{x_t\}$ only every m-th period, the resulting process $\{\tilde{x}_t\}, t = m, 2m, 3m, \ldots$ is said to be temporally aggregated. The properties of temporally aggregated ARFIMA processes are derived in Baillie, Nijman, and Tschernig (1994). They show that the degree of fractional integration is independent of temporal aggregation if $d \geq 0$ in case of stock variables and $d \geq -1$ in case of flow variables. Although the class of ARFIMA(p, d, q) models is not closed under temporal aggregation, a temporally aggregated process may well be approximated by an ARFIMA(p, d, q^*) model with a low order MA polynomial. Furthermore, they provide a numerical procedure to calculate the q^* MA parameters of the temporally aggregated, approximate ARFIMA(p, d, q^*) process for given values of $\alpha(B)$, d, and $\beta(B)$ and a given order of temporal aggregation m. As this procedure is applied in the empirical analysis it is briefly described in appendix A.

It is well documented that high frequency data of financial time series exhibit conditional heteroskedasticity. To allow for this in the framework of ARFIMA models, the ARFIMA(p,d,q)-GARCH(P,Q) model is applied which was introduced by Baillie, Chung, and Tieslau (1992). It replaces (2) by a white noise process with conditional variance

$$\{\varepsilon_t\} \sim WN(0, \sigma_t^2). \tag{8}$$

The conditional variance σ_t^2 is determined by the lagged values of the conditional white noise variance and lagged squared realisations of the error term ε_t^2

$$\phi(B)\sigma_t^2 = \xi + \theta(B)\varepsilon_t^2. \tag{9}$$

with the polynomials defined by $\phi(B) = 1 - \phi_1 B - \ldots - \phi_P B^P$ and $\theta(B) = \theta_1 B + \ldots + \theta_Q B^Q$. Since ε_t^2 cannot be negative, sufficient conditions to ensure nonnegativity require that $\phi_i \geq 0, i = 1, \ldots, P$, $\theta_i \geq 0, i = 1, \ldots, Q$, $\xi > 0$ and $\phi(1) > 0$. Given the nonnegativity restriction, ε_t^2 is covariance-stationary if

$$-\phi(1) + \theta(1) < 0 \quad \text{or} \tag{10}$$

$$\phi_1 + \phi_2 + \ldots + \phi_P + \theta_1 + \theta_2 + \ldots + \theta_Q < 1 \tag{11}$$

(see Hamilton (1994, p. 665-6) for details).

By scaling the errors ε_t by the conditional standard deviation σ_t one again obtains homoskedastic errors

$$\nu_t = \frac{\varepsilon_t}{\sigma_t} \tag{12}$$

that are i.i.d with zero mean and unit variance.

3 EMPIRICAL RESULTS

This section re-examines the evidence of long memory in foreign exchange rates. It presents estimation and prediction results on daily, weekly, monthly, and quarterly changes of the US-Dollar spot rates of the Deutsche Mark (DM) and the Swiss Franc (SF) as well as of the changes of the Deutsche Mark/Swiss Franc spot rate. The total period under investigation runs from January 31, 1973 until April 30, 1990. The results of different observation frequencies are evaluated in light of the properties of temporal aggregation which were presented in section 2.

The issue of structural stability over time of the selected models is addressed by estimating different subperiods. In order to evaluate the importance of possible adjustment effects after the end of Bretton-Woods, one subperiod starts on January 1, 1976. As the end of the seventies was marked by a considerable change in the US monetary policy and the beginning of the European Monetary System, the total sample was also divided into two subsamples covering January 1, 1973 until December 31, 1979 and January 1, 1980 until April 31, 1990, respectively.

3.1 Data and methods

The DM and SF dollar spot rates underlying this study have kindly been made available by the Bank for International Settlements in Basle, Switzerland. They correspond to the official fixing at 1 p.m. Frankfurt time. Prior to 1977 the SF/US-Dollar spot rate conforms to the market closing middle rate. Subsequently, the official base rate at 1 p.m. Swiss time is used. The DM/SF spot rate is obtained by calculating the cross rate of the DM/US-Dollar and SF/US-Dollar. Weekly data are sampled by using end-of-week quotations. By taking observations of the last trading day of each month, the monthly series are retrieved. Finally, the quarterly series consist of the last quotation in each quarter. As usual, the first differences of the logarithmized values are taken for estimation.

For estimation only approximate maximum likelihood methods are used. While for estimating ordinary ARFIMA(p, d, q) specifications two different approximate frequency domain maximum likelihood methods are employed, namely the Whittle estimator (24) or its approximation (27), the approximate time domain maximum likelihood method (29) of Baillie, Chung, and Tieslau (1992) is applied for estimating the more complex ARFIMA(p, d, q)-GARCH(P, Q) models. All three estimation methods are described in appendix B.

Since it is well known that daily changes of foreign exchange rates exhibit conditional heteroskedasticity (e.g. Hsieh (1989), Bollerslev and Engle (1993)), it seems appropriate to apply the approximate time domain maximum likelihood method (29) that allows the estimation of ARFIMA-GARCH specifications. However, in case of more than 4000 observations the calculation of the corresponding approximate maximum likelihood (29) is very computer intensive. On the other hand, given such large number of observations one may well ignore the GARCH structure for the estimation of the ARFIMA parameters. In case of an AR-GARCH model Bollerslev (1986) shows that the AR parameters can be estimated consistently and without loss of asymptotic efficiency when the GARCH structure is ignored. It can be conjectured that this result remains valid even for general ARFIMA specifications if the ARFIMA model is invertible such that it has an infinite AR representation. As an illustration an ARFIMA(2,d,0)-ARCH(1) process with 4000 observations is generated and estimated with the approximate Whittle estimator (27). Setting $\alpha_1 = -.2$, $\alpha_2 = -.2$, $d = .1$, $\xi = 0.5$ and $\phi_1 = .2$ one obtains estimates with standard errors smaller than 0.02 and point estimates that are very close (< 0.02) to the true values. For this reason, the well known GARCH structure is neglected in the estimation of daily data and the approximate Whittle estimator (27) is applied instead.

For the analysis of weekly and monthly data, however, the approximate time domain method for the simultaneous estimation of ARFIMA-GARCH models (29) is used. From various Monte Carlo simulations it is known that detecting long memory poses severe problems if the sample size becomes small, e.g. consists of 100 observations or less. Thus, there is only a small probability for finding evidence for long memory in seventeen years of quarterly data even it is present. This probability can only be slightly increased by using the Whittle estimator (24) which is therefore employed instead of its approximation (27) (cf. Tschernig (1994)). This is possible since conditional heteroskedasticity is negligible in quarterly data.

When no GARCH errors are considered, the model selection is based on a set of 17 alternatives of ARFIMA$(p, 0, q)$ and ARFIMA(p, d, q) specifications with the length of the AR and the MA polynomial varying between 0 and 2. If in addition GARCH errors are allowed for, all 17 ARFIMA(p, d, q) specifications are estimated for GARCH(0,0), GARCH(1,0) and GARCH(1,1) disturbances. For model selection, the AIC and Schwarz criterion are used. In case of several hundred observations, the model selection turns out to be very computer intensive if all ARFIMA-GARCH specifications are considered as described above. Therefore, the model selection procedure is only carried out for full samples. The selected model specification is then also used for each subperiod.

Out-of-sample forecasts provide the most rigorous test of a selected and estimated time series model. Therefore, out-of-sample forecasts for horizons of six and twelve months are conducted. Following the argument of Engel and Hamilton (1990), the random walk model with drift is employed as a benchmark. Using conventional statistical tests they find that the null hypothesis of a constant drift term over their estimation and prediction period is rejected and conclude that this implicitly rejects both random walk hypotheses, with and without drift, although the latter outperforms the former for their period of analysis. For quarterly data the predictions are carried out by means of the exact prediction method using the Innovations Algorithm of Brockwell and Davis (1991). All other predictions are computed with the approximate method ((34) and (35)). Both methods are described briefly in appendix B. The forecasting performance of each model is evaluated by the mean squared error (MSE)

$$MSE = \sum_{i=T}^{\bar{T}-h} \frac{(\hat{s}_{i+h} - s_{i+h})^2}{\bar{T} - T - h} \tag{13}$$

where T corresponds to the minimal length of the estimation period. $\bar{T}$ denotes the total number of periods investigated including the last predicted date. h represents the number of forecasted periods, and $\hat{s}_{t+h}$ and s_{t+h} denote the predicted and the actual logarithmized foreign exchange rate at time $t + h$, respectively. Following Engel and Hamilton (1990) the initial predictions $\hat{s}_{T+h}$ are based on the estimation period from January 1, 1973 until December 31, 1983. Then, the out-of-sample forecasts $\hat{s}_{t+h}$ are repeated with the estimation period being each time extended by one period until the predictions reach January 1, 1988.

3.2 DM/US-Dollar spot exchange rates

The results for daily changes in the DM/US-Dollar spot rate for the total period as well as for the three subperiods are shown in table 1. The memory parameter d is always significant and positive if the approximate Whittle estimator and the AIC criterion are employed. Note that the absolute values of the estimated short memory AR parameters closely resemble the first two parameters of the infinite AR representation of an ARFIMA$(0, d, 0)$ process with identical d. However, there do never exist any common roots between the long and the short memory components of an ARFIMA(p, d, q) process as the ARFIMA$(0, d, 0)$ process parallels either an infinite MA or an infinite AR process which cannot be described by a finite ARMA process. Thus, finding the p or q estimated short memory parameters coinciding with the first p or q parameters of the infinite MA or AR representation of the ARFIMA$(0, d, 0)$ process might indicate that there is only weak stochastic dependence between narrow observations while stochastic dependence remains non-negligible between distant observations. Indeed, the parameters of the infinite AR representation of the estimated ARFIMA(2,d,0) process and the parameters of the infinite AR representation of an ARFIMA(0,d,0) process with identical d differ after the fourth lag only in the fifth digit after the decimal point. There is almost no variation in the estimates of d across the three subperiods. Table 1 shows that the estimate of the memory parameter only slightly changes if the early years of the seventies are ignored. Thus, there does not seem to be evidence for structural change.

insert table 1 here

In contrast to the AIC, the Schwarz criterion selects for the full period an MA(1) process with an insignificant parameter estimate very close to zero. From Monte Carlo studies it is known that the Schwarz criterion may behave comparatively poorly to the AIC when the true process is an ARFIMA(p, d, q) process and the sample size is small (cf. Schmidt and Tschernig (1994)). As this result hardly holds for very large samples, one should choose the MA(1) model selected by the Schwarz criterion if one aims at avoiding inconsistent model selection, a property inherent to the AIC. Then, daily DM/US-Dollar rate changes would be uncorrelated.

Before any final conclusions were drawn, properties of temporally aggregated ARFIMA processes should be exploited. From section 2 it is known that the memory parameter d remains unchanged if the process exhibits long memory and the observation frequency is decreased. However, the short memory components change. In case of an ARFIMA$(2, d, 0)$ process temporal aggregation yields an ARFIMA$(2, d, \infty)$ process. Table 2 shows the values of some short memory parameters of the temporally aggregated processes that are computed with the method described in appendix A. It can be seen that both the AR parameters as well as the first two MA parameters are zero or almost zero for all temporally aggregated processes under consideration. Therefore, all temporally aggregated processes can be well approximated by an ARFIMA$(0, d, 0)$ process. If the true process is a pure long memory process, Schmidt and Tschernig (1994) find in their Monte Carlo simulations that the Schwarz criterion is more likely to select the true ARFIMA$(0, d, 0)$ process than the AIC if the sample size is small or medium. This is in contrast to the case of the general ARFIMA(p, d, q) process considered above. As the working hypothesis is that the ARFIMA$(2, d, 0)$ model is correct, the Schwarz criterion is employed in the sequel. In case the working hypothesis is false this should be indicated by point estimates of d which are hardly compatible with the predictions of temporal aggregation.

insert table 2 here

When one estimates weekly changes of the DM/US-Dollar rate over the full period, one almost precisely obtains the ARFIMA$(0, d, 0)$ process with $d = 0.079$ which is produced by temporal aggregation. The parameter estimates of an ARFIMA$(0, d, 0)$-GARCH(1,1) process which is selected out of 51 different ARFIMA-GARCH specifications by the Schwarz criterion are shown in the second column of table 3. It should be noted that ignoring the highly significant conditional heteroskedasticity does not result in a much different memory parameter estimate ($d = 0.074$ compared to $d = 0.063$) while the standard error is identical in both cases. Tschernig (1994) contains detailed results based on ARFIMA(p, d, q)

specifications that are estimated with the approximate Whittle estimator.

insert table 3 here

In contrast to daily data, there is more variation of the memory parameter estimates for weekly data across the three subperiods. This can be seen in columns three to five of table 3. Ignoring the early years of the seventies makes the d estimate insignificant, while ignoring the eighties leads to an increase in $\hat{d}$. Nevertheless, the qualitative behavior resembles the findings for the daily changes. Thus, one might explain the variation in the parameter estimates more on purely statistical grounds than by structural change although there is no reliable test for structural change in the presence of long memory. Note that a possible explanation of the latter could be learning and adjustment effects that followed the collapse of the Bretton-Woods system.

When one considers monthly data, once again a pure long memory specification is selected for the total period if the Schwarz criterion is used. Table 4 displays the results of the ARFIMA$(0, d, 0)$ models for all four periods. Although now the memory parameter estimates are no longer significant, they lie in a reasonable range to be in line with the predictions of temporal aggregation except for the seventies when the sign changes. One should have in mind, however, that the sample of monthly changes in the eighties consists of only 122 observations, a number which is known to be at the lower limit for long memory analysis. For this reason it is not very informative to investigate quarterly data with a maximum of 68 observations. For completeness, the results are shown in table 5. The Schwarz criterion selects an ARFIMA$(0, d, 0)$ model when the Whittle estimator (24) is applied. In contrast to previous observation frequencies the memory parameter estimate is now close to zero, insignificant and no longer supportive to the hypothesis of temporal aggregation.

insert table 4 here

insert table 5 here

In sum, there is substantial evidence for weak long memory in the changes of the DM/US-Dollar spot rate. In particular, both, the selected models as their parameter estimates across various observation frequencies in general correspond to the prediction of temporal aggregation which are based on the ARFIMA$(2, d, 0)$ specification of daily changes. In some cases, however, there is a slight variation in the point estimates and their significance across different subperiods and observation frequencies.

This evidence pro long memory is also supported by out-of-sample forecasts. Table 6 shows the relative improvement or deterioration of the MSE of the ARFIMA$(0, d, 0)$ specification to the MSE of the random walk with drift for a 6- and a 12-month forecast horizon for weekly, monthly and quarterly exchange rate changes. Except for the highly unreliable results of quarterly data, all ARFIMA predictions show a slight improvement over the random walk with drift model. Note that the estimated d for daily changes is too small in order to produce forecasts which differ noticeably from the mean of the time series. Therefore there are no out-of-sample forecasts presented for daily data.

insert table 6 here

3.3 SF/US-Dollar spot exchange rates

The daily changes of the SF/US-Dollar exchange rates are estimated using the approximate Whittle estimator for reasons explained in subsection 3.1. Selecting the model specification on the basis of the AIC, one obtains the same ARFIMA$(2, d, 0)$ specification for the total period as for the daily changes of the DM/US-Dollar rates. However, as can be seen from table 7 the magnitude as well as the t-values of the parameter estimates are smaller than for the DM/US-Dollar series. This will also be the final result of this subsection. The SF/US-Dollar spot rate behaves similarly to the DM/US-Dollar spot rate but its stochastic structure is weaker.

insert table 7 here

The memory parameter estimates for the ARFIMA$(2, d, 0)$ specification vary more across the subperiods than those of the DM/US-Dollar rate. They are significant at the 5% level only for the total period and the eighties. Not surprisingly, when the model selection procedure is carried out for each subperiod, the AIC chooses ARFIMA specifications only for these periods (cf. Tschernig (1994, p. 194, Tabelle 7.14)).

Employing the Schwarz criterion one obtains an insignificant MA(1) process for the total period, just like for the DM/US-Dollar rate. Again, properties of temporal aggregation are used to resolve the contradicting model selection results. Table 2 shows parameter values of temporally aggregated ARFIMA$(2, d, 2)$ processes of the ARFIMA$(2, d, 0)$ process which was chosen by the AIC for daily data. As in the previous section the short memory parameters are around zero so that temporal aggregation results in an ARFIMA$(0, d, 0)$ process. Following the arguments in the previous subsection, the Schwarz criterion is therefore used for model selection of temporally aggregated time series. The approximate time domain maximum likelihood ARFIMA(p, d, q)-GARCH(P, Q) estimates for weekly changes are contained in Table 8. For the full period the selected ARFIMA$(0, d, 0)$ model as well as the memory estimate of $\hat{d} = 0.049$ correspond well to the theoretical values computed in table 2. Furthermore, the variation across subperiods which is observed for daily changes also carries over to weekly data. Note that the significant d estimate for the changes in the seventies of 0.117 is twice as large as for the total period and five times as large as the corresponding value for daily changes.

insert table 8 here

For monthly changes of the SF/US-Dollar spot rate the Schwarz criterion also selects the ARFIMA$(0, d, 0)$ specification with memory parameter estimates which are compatible with temporal aggregation except for the seventies. This can be seen from table 9. The memory parameter estimates are not significant, however. These results were already found for the monthly changes of the DM/US-Dollar rates in the previous subsection. As mentioned there, the insignificance may be caused by the few number of observations. Finally, the Schwarz criterion selects a short memory MA(1) process for quarterly observations as can be seen from table 5.

insert table 9 here

In sum, there is also evidence for weak long memory in the changes of the SF/US-Dollar spot rates although it is weaker than for the changes of the DM/US-Dollar spot rates. In particular, there is more variation across subperiods and the estimates exhibit in general smaller t-values and are less often significant. Nevertheless, the third and fourth row of table 6 show that there is some improvement of the relative out-of-sample performance of the ARFIMA$(0, d, 0)$ model over the random walk with drift. It is, however, smaller than in case of the DM/US-Dollar changes.

3.4 DM/SF spot exchange rates

Is there also evidence for long memory in exchange rates which do not involve the US-Dollar spot rate? In order to obtain a first answer to this question, the DM/SF spot rate is investigated. It is chosen because the Swiss Franc is not a member of the EMS while still being an important international currency. For the total period of daily changes both selection criteria, the AIC as well as the

Schwarz criterion select an ARMA(2,2) model with highly significant parameter estimates as shown in the second column of table 10. The estimates are obtained using the approximate Whittle estimator. However, when this specification is estimated for the subperiods from 1976 to 1990 and 1973 to 1980, the Hessian matrix cannot be inverted. Thus, it is necessary to conduct the model selection procedure for each subperiod and therefore the results of Tschernig (1994, p. 196, Tabelle 7.15) are reported here. Table 10 exhibits the estimated models chosen by the AIC. For the subperiod 1976 to 1990 one obtains an ARFIMA($0, d, 0$) model with an insignificant memory parameter estimate close to zero. For the period 1980 to 1990, the Schwarz criterion produces the same result while the AIC chooses the ARMA(2,2) specification as for the total period. Now, however, its highly significant estimates are close to having common roots. As a result, there is no evidence for any stochastic structure in the subperiods covering the eighties. This finding can be interpreted as evidence for structural change. Indeed, when the eighties are ignored, either a significant AR(2) or a significant MA(1) model is selected depending on the choice of the selection criterion. One reason for structural instability could be that the Swiss central bank started to link the Swiss Franc to the EMS at the end of the seventies. Thus, there is no indication for long memory at all.

insert table 10 here

When weekly data are considered and ARFIMA(p, d, q)-GARCH(P, Q) specifications are allowed for in the estimation procedure, both selection criteria select a specification which involves a long memory component. While an ARFIMA$(1, d, 1)$-GARCH(1,1) specification with a significant memory estimate of $\hat{d} = -.025$ is selected by the AIC, column two of table 11 shows an insignificant ARFIMA$(0, d, 0)$-GARCH(1,1) model chosen by the Schwarz criterion. Independent of the choice of the selection criterion, the memory parameter estimate is close to zero. Applying the latter specification in the analysis of the subperiods, the conclusion drawn from the analysis of daily data is confirmed. The subperiod of the seventies appears to exhibit stochastic structure in contrast to the eighties. However, one should not interpret the point estimate of $\hat{d} = 0.075$ of column five in favor of the presence of long memory as one should expect a complete model selection procedure to choose a different model specification since a long memory model can never be the result of temporal aggregation when the daily changes do not exhibit long memory.

insert table 11 here

Both, the AIC as well as the Schwarz criterion select an AR(1)-ARCH(1) specification for the full period of monthly changes. As table 12 shows, the AR parameter is insignificant for the full period and all subperiods. Note that now the seventies do no longer show any sign of stochastic structure. When the quarterly changes are investigated with the Whittle estimator, one finds an insignificant MA(1) process as shown in table 5.

insert table 12 here

Therefore one can conclude that there is no long memory present in the changes of the European DM/SF spot rate. For this reason, no out-of-sample forecasts are carried out. There is, however, evidence that the highfrequent changes show a short memory structure in the seventies.

Taking these results on the changes of the DM/US-Dollar, the SF/US-Dollar, and the DM/SF exchange rate together, one comes to the conclusion that long memory is solely a phenomenon linked to the US-Dollar exchange rates and therefore possibly to the US economy, while the increments of the European DM/SF rate show short memory behavior in the seventies. Several theoretical explanations of long memory in foreign exchange rates have been put forward in the literature. Most notably, Cheung (1993) suggests that the dynamics of foreign exchange rates are tied to the behavior of the relative national prices if the purchasing power parity holds. He also presents some empirical evidence for long memory in the monthly changes of relative national consumer price indices. Furthermore, he considers the role of various fundamentals such as relative money supplies or relative outputs as they are standard determinants in exchange rate models. In Tschernig (1994) this line of thought is taken up. It is shown for an "example economy" based on the Lucas (1982) two country–two goods model that the foreign exchange rate exhibits the degree of (possibly fractional) integration which ranks highest among the various degrees of (possibly fractional) integration of the national good and money markets. In this modelling approach expectations in the determination of exchange rates are neglected. However, as Tschernig (1994) has shown this result also holds in a model of the typical asset market approach introduced by Mussa (1976) or Frenkel and Mussa (1980). The asset market model considered in Tschernig (1994) is based on a simple monetary model, the assumption of purchasing power parity, and the uncovered interest rate parity. Finally, it should be pointed out that long memory in foreign exchange rates may also be the result of market inefficiencies.

4 CONCLUSION

As the existing evidence on long memory in foreign exchange rates is partially inconclusive, this paper attempted a more detailed analysis of three foreign exchange rates regarding the presence of long memory captured by the fractional ARIMA(p, d, q) model. This linear time series model is a direct generalization of the ARIMA(p, d, q) model since it allows the differencing parameter d to take real values. This paper extended previous investigations in several directions. First, results on the temporal aggregation of ARFIMA(p, d, q) processes were used in order to check the existing evidence for long memory in DM/US-Dollar and SF/US-Dollar rates. For this reason daily, weekly, monthly and quarterly data were considered. Second, in order to address the issue of structural stability, three subperiods were investigated in addition to the total period of 1973 to 1990. Third, the analysis also includes the European DM/SF spot rate in order to check the dependence of the previous evidence on long memory on the US economy. Fourth, when necessary, an estimation procedure is employed that allows to estimate ARFIMA(p, d, q) specifications with GARCH(P, Q) errors.

The strongest evidence for weak long memory was found in the changes of the DM/US-Dollar spot rates followed by the changes of the SF/US-Dollar spot rates where the stochastic structure of both dollar spot rates appears to be quite similar although the memory parameter estimates for the changes of the SF/US-Dollar spot rate show more variation across the subperiods that were investigated. Daily exchange rate changes are well described by ARFIMA$(2, d, 0)$ processes with a small and positive memory parameter. The AR component basically disappears if the observation frequency is reduced to weekly, monthly or quarterly data while the memory parameter remains the same. This can be shown by using results of temporal aggregation of ARFIMA processes. The empirical results for weekly, monthly, and quarterly changes are found to be generally consistent with the properties of temporal aggregation. However, the parameter estimates for monthly and quarterly data are no longer significant. This may be attributed to the insufficient number of observation.

In contrast, there is no evidence for long memory in the DM/SF spot rate changes. This time series is found to exhibit short memory only for the seventies. These findings might indicate that the presence of long memory is closely linked to the behavior of the US currency. In particular they may illustrate the importance of the behavior of the national monetary authorities like the Federal Reserve Bank.

APPENDIX

A TEMPORAL AGGREGATION

This section reports results of Baillie, Nijman, and Tschernig (1994) on the temporal aggregation of ARFIMA(p, d, q) processes which are used in section 3.

Let $\{x_t\}$ denote an ARFIMA$(2, d, 0)$ process

$$(1 - \alpha_1 B - \alpha_2 B^2)(1 - B)^d x_t = \varepsilon_t, \quad \varepsilon_t \sim WN(0, \sigma_\varepsilon^2). \tag{14}$$

If one can observe x_t only every m-th period, $t = m, 2m, 3m, \ldots$, the corresponding temporally aggregated process $\{\tilde{x}_t\}$ is denoted by

$$\tilde{x}_t = w(B)x_t, \quad t = m, 2m, 3m, \ldots \tag{15}$$

with

$$w(B) = \begin{cases} 1 & \text{in case of stock variables} \\ \sum_{i=0}^{m-1} B^i & \text{in case of flow variables} \end{cases} \tag{16}$$

where B denotes the backshift operator. In order to obtain a representation of the temporally aggregated process, multiply (14) by $w(B)$, $(1 + B + \ldots + B^{m-1})^d$ and

$$(1 + a_1 B + \ldots + a_1^{m-1} B^{m-1})(1 + a_2 B + \ldots + a_2^{m-1} B^{m-1})$$

where a_1 and a_2 denote the (possibly complex) roots of the AR(2) polynomial. Then one obtains

$$(1 - a_1^m B^m)(1 - a_2^m B^m)(1 - B^m)^d \tilde{x}_t = e_t, \quad t = m, 2m, 3m, \ldots \tag{17}$$

with

$$
\begin{aligned}
e_t &= (1 + a_1 B + \ldots + a_1^{m-1} B^{m-1})(1 + a_2 B + \ldots + a_2^{m-1} B^{m-1}) \qquad (18) \\
&\quad (1 + B + \ldots + B^{m-1})^d w(B) \varepsilon_t, \quad t = 1, 2, \ldots.
\end{aligned}
$$

From (18) it can be seen that the process $\{e_t\}, t = m, 2m, 3m, \ldots$ is not a finite MA process if $d \neq 0$ since the power series expansion of $(1 + B + \ldots + B^{m-1})^d$ is infinite for noninteger d (see section 2). In the latter case e_t is also influenced by lagged values $\varepsilon_{t-i}, i \geq m$ since its MA representation is infinite. Therefore $\{\tilde{x}_t\}$ is no longer an ARFIMA$(2, d, 0)$ process but an ARFIMA$(2, d, \infty)$ process. Therefore, the class of ARFIMA(p, d, q) processes is not closed with respect to temporal aggregation.

However, Baillie, Nijman, and Tschernig (1994) show that the order of fractional integration remains the same if $d \geq 0$ in case of stock variables and $d \geq -1$ in case of flow variables. Furthermore, they show that if these conditions are fulfilled it is possible to approximate the ARFIMA(p, d, ∞) process by an ARFIMA(p, d, q) process with small q.

The values of the approximating MA(q) process have to be computed numerically by solving the equation system

$$
\frac{\sum_{i=0}^{q-j} \beta_i \beta_{i+j}}{\sum_{i=0}^{q} \beta_i^2} = \rho(jm), \quad j = 1, 2, \ldots, q. \qquad (19)
$$

where $\rho(jm) = \gamma(jm)/\gamma(0)$ and $\gamma(jm)$ denote the autocorrelation function and autocovariance function for lag $\tau = jm$, respectively. One then chooses a set of solutions that fulfils the conditions of invertibility. The autocovariances are calculated by means of numerical integration from the well-known relationship between the autocovariance function and the spectral density function

$$\gamma(\tau) = \int_{-\pi}^{\pi} f_e(\omega) e^{-i\tau\omega} d\omega \tag{20}$$

where the spectral density function of $\{e_t\}$ in case of an ARFIMA$(2, d, 0)$ process is given by

$$\begin{aligned} f_e(\omega) &= |1 + a_1 e^{-i\omega} + \ldots + a_1^{m-1} e^{-i(m-1)\omega}|^2 |1 + a_2 e^{-i\omega} + \ldots + a_2^{m-1} e^{-i(m-1)\omega}|^2 \\ & |1 + e^{-i\omega} + \ldots + e^{-i(m-1)\omega}|^2 f_\varepsilon(\omega). \end{aligned} \tag{21}$$

$f_\varepsilon(\omega) = \frac{1}{2\pi}\sigma^2$ denotes the spectral density of the white noise process $\{\varepsilon_t\}$.

If one computes the autocorrelations of the temporally aggregated MA process $\{e_t\}, t = m, 2m, \ldots$, it can be seen that they tend quickly to zero independently of d if the underlying process is an ARFIMA$(0, d, 0)$ process (see Baillie, Nijman, and Tschernig (1994) for numerical examples). If an AR component is present, then the fast convergence may start at higher lags. It is this quick convergence that allows the approximation of the infinite MA process (18) by a low order MA(q) process.

B ESTIMATING ARFIMA MODELS AND METHODS FOR PREDICTION

This section summarizes the estimation and prediction methods as well as their properties for ARFIMA(p, d, q)-GARCH(P, Q) specifications underlying the empirical analysis in section 3.

B.1 Estimation

ARFIMA(p, d, q) models. Several methods have been proposed to estimate the parameters of an ARFIMA(p, d, q) model from a given series of observations: most notably the maximization of the exact likelihood in the time domain and the maximization of several variants of approximate likelihood functions in the frequency domain. Although one might expect that among maximum likelihood methods the exact maximum likelihood method to perform best, this is not necessarily true if the mean has to be estimated and long memory is present. This is because the exact time domain method depends on the estimate of the mean which converges with a slower rate than with $\sqrt{T}$ to its true value if the stochastic process exhibits long memory. In contrast, approximate frequency domain methods are independent of the mean estimation. For this reason the latter methods perform equally well in most cases but are less computer intensive. This is the result of a detailed Monte Carlo simulation of Cheung and Diebold (1994).

Two approximate frequency domain maximum likelihood methods are employed in this analysis, the Whittle estimator and its approximation (Whittle (1951)). They will be briefly described in this section. Let $\lambda = (\alpha', d, \beta')'$ denote the parameter vector which is to be estimated. Following the presentation of Fox and Taqqu (1986), the key element of both methods is the approximation of the inverted covariance matrix $\Sigma^{-1}(\lambda)$ of the stochastic process by an expression in the frequency domain $A_T(\lambda)$ where each element $[A_T(\lambda)]_{jk}$ is given by

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{g(\omega;\lambda)}e^{i(j-k)\omega}d\omega$$

and $g(\omega;\lambda)=\frac{2\pi}{\sigma^2}f(\omega;\lambda)$. Assuming normality and approximating $\det(\Sigma)\sim(\sigma^2)^T$, minimization of the likelihood function leads to the estimator of the white noise variance

$$\tilde{\sigma}_T^2=\frac{(x-\mu)'A_T(\lambda)(x-\mu)}{T} \tag{22}$$

where x and μ denote the vector of T observations and the true mean, respectively. Minimizing (22) then gives the parameter estimates $\tilde{\lambda}$. If one further uses the empirical autocovariance function

$$\tilde{\gamma}(\tau)=\frac{1}{T}\sum_{t=1}^{T-|\tau|}x(t)x(t+|\tau|), \tag{23}$$

one obtains

$$\tilde{\sigma}_T^2(\lambda)=\sum_{\tau=-T+1}^{T-1}\frac{1}{2\pi}\tilde{\gamma}(\tau)\int_{-\pi}^{\pi}\frac{1}{g(\omega;\lambda)}e^{-i\tau\omega}d\omega. \tag{24}$$

On the basis of the periodogram

$$I(\omega)=\frac{1}{2\pi}\sum_{\tau=-T+1}^{T-1}\tilde{\gamma}(\tau)e^{-\tau\omega}. \tag{25}$$

an alternative representation of (24) is given by

$$\tilde{\sigma}_T^2(\lambda)=\int_{-\pi}^{\pi}\frac{I(\omega)}{g(\omega;\lambda)}d\omega. \tag{26}$$

which is known as the Whittle estimator. Dahlhaus (1989) shows its asymptotic normality and efficiency.

However, as the numerical integration of the fraction containing the periodogram in (26) is cumbersome and possibly inaccurate, Tschernig (1994) suggests an alternative computation method of the Whittle estimator that avoids the integration of the periodogram. It is based on the representation (24) which allows to make use of Sowell's (1992) method to compute the autocovariance function of an ARFIMA(p,d,q) process. Although this method is exact, this algorithm still requires the computation of various hypergeometric functions if the MA-part is not zero.

In case of larger data sets a faster solution to the integration problem in (24) is to approximate the integral by the sum over the Fourier frequencies $\omega_u = 2\pi u/T$, $u = 1, \ldots, T-1$. This leads to the widely used approximate Whittle estimator

$$\hat{\sigma}_T^2(\lambda) = \frac{2\pi}{T} \sum_{u=1}^{T-1} \frac{I_T(\omega_u)}{g(\omega_u; \lambda)}. \tag{27}$$

For this estimator Robinson (1990) sketches a way to prove asymptotic efficiency.

To speed up the calculation of the asymptotic covariance matrix $H(\lambda_0)$ of the parameter estimates, Cheung (1993) proposes the approximation

$$2\hat{\sigma}_T^2(\lambda) \left[\frac{\partial^2 \sigma_T^2(\lambda)}{\partial \lambda' \partial \lambda} |_{\bar{\lambda}_0} \right]^{-1} \longrightarrow H(\lambda_0). \tag{28}$$

For model selection, the Akaike Information Criterion (AIC) and the Schwarz criterion are applied. Concerning the identification of pure long memory processes, Schmidt and Tschernig (1994) show that using the Schwarz criterion implies the highest probability to select the true process if the sample size is small. They also find that reliable identification of a weak pure long memory process requires several hundred observations.

Unfortunately, the Schwarz criterion is no longer the optimal choice when long and short memory components are mixed in an ARFIMA(p, d, q) process as it punishes overparameterization quite heavily as compared, for instance, with the AIC. Schmidt and Tschernig (1994) illustrate this point in a Monte Carlo example using several selected ARFIMA(p, d, q) specifications that are particularly unfavorable to correct identification. In these cases using the AIC leads to the best results. Thus, both criteria are applied in this study.

ARFIMA(p, d, q)-GARCH(P, Q) models. In the presence of conditionally heteroskedastic errors ε_t it is no longer feasible to approximate the likelihood function in the frequency domain. In this case we follow the approximate maximum likelihood approach suggested by Baillie, Chung, and Tieslau (1992). Assuming conditional normality and neglecting starting values one obtains the approximate log likelihood

$$\ln l(\lambda|x) = -\frac{1}{2T}\sum(\ln\sigma_t^2 + \frac{\varepsilon_t^2}{\sigma_t^2}) \tag{29}$$

where the parameter vector λ is given by $\lambda = (\mu, \alpha', d, \beta', \xi, \phi', \theta')'$. Baillie, Chung, and Tieslau (1992) state that for stationary and invertible ARFIMA-GARCH processes (1), (8), (9) all parameter estimates will be consistent with normal convergence rates. For the mean μ, however, the convergence rate is $T^{0.5-d}$. Chung and Baillie (1992) provide some simulation evidence that this method works well for ARFIMA models with at least 100 observations. For model selection the Akaike Information and Schwarz criteria are applied.

B.2 Prediction

Depending on the number of observations either an exact or an approximate prediction method is employed for the empirical analysis of foreign exchange rates in section 3. Prediction theory for ARFIMA(p, d, q) processes is directly available as they belong to the class of linear stochastic processes. Given processes with finite variance, the best predictor is usually defined as the predictor with minimal mean squared error

$$MSE[\hat{X}_{t+h}] = E[(X_{t+h} - \hat{X}_{t+h})^2] \tag{30}$$

where $\hat{X}_{t+h}$ denotes the prediction of the random variable X_{t+h}. Due to the linearity of the ARFIMA process, the best predictor is linear. It is given by

$$\hat{X}_{t+h} = \sum_{i=1}^{t} \theta_{ti}^{(h)} x_{t-i} \tag{31}$$

where the parameters $\theta_{ti}^{(h)}$ are obtained from the linear equation system

$$\sum_{j=1}^{t} \theta_{tj}^{(h)} \gamma(i-j) = \gamma(h+i-j), \quad i = 1, \ldots, t \tag{32}$$

and the autocovariances of an ARFIMA(p, d, q) model can be computed by means of Sowell's (1992) algorithm. As solving the linear equation system (32) becomes cumbersome for large $t+h$, an attractive alternative for calculating the predictions $\hat{X}_{t+h}$ is given by the Innovations Algorithm of Brockwell and Davis (1991, pp. 172 – 173). Their approach is used in the current study for prediction in small samples.

For predicting ARFIMA processes, the calculation of their autocovariance function can be avoided if the infinite AR representation

$$X_t = \psi(L)X_t + \varepsilon_t \tag{33}$$

$$\psi(L) = \alpha^{-1}(L)\beta(L)\nabla^{-d} \tag{34}$$

of the ARFIMA(p,d,q) process (1) is used with all $X_j, j \leq 0$, set to zero. Then, the approximate predictor of x_{t+h} is given by

$$\tilde{X}_{t+h} = \sum_{j=1}^{t+h-1} \psi_{t+h-j}\tilde{X}_j \tag{35}$$

and $\tilde{X}_j = x_j$ for $j \leq t$ (cf. Brockwell and Davis (1991, pp. 183 and 533).

REFERENCES

Baillie, Richard T., Chung, Ching-Fan, and Tieslau, Margie A.. "The long memory and variability of inflation: a reappraisal of the Friedman Hypothesis.", *Econometrics and Economic Theory Paper*, No. 9102, Department of Economics, Michigan State University, 1992.

Baillie, Richard T., Nijman, Theo, and Tschernig, Rolf. "Temporal aggregation of fractionally integrated ARMA models." manuscript, 1994.

Bollerslev, Tim. "Generalized Autoregressive Conditional Heteroskedasticity.", *Journal of Econometrics*, 31 (1986): 307-327.

Bollerslev, Tim and Engle, Robert F.. "Common persistence in conditional variances.", *Econometrica*, 61 (January 1993): 167 – 186.

Booth, G.G., Kaen, F.R., and Koveos, P.E.. "R/S analysis of foreign exchange rates under two international monetary regimes.", *Journal of Monetary Economics*, 10 (1982): 407–15.

Brockwell, Peter J. and Davis, Richard A.. *Time Series: Theory and Methods.* New York et al., Springer Verlag, 1991.

Cheung, Yin-Wong. "Long memory in foreign-exchange rates.", *Journal of Business and Economic Statistics*, 11 (January 1993): 93 – 101.

Cheung, Yin-Wong and Diebold, Francis X.. "On maximum-likelihood estimation of the differencing parameter of fractionally integrated noise with unknown mean.", *Journal of Econometrics*, 62 (1994): 301 – 316.

Chung, Ching-Fan and Baillie, Richard T.. "Small sample properties of estimators of ARFIMA models.", manuscript, Michigan State University, 1992.

Cornell, B. and Dietrich, J.K.. "The efficiency of the market for foreign exchange under floating exchange rates.", *Review of Economics and Statistics*, (1978): 111 – 120.

Dahlhaus, Rainer. "Efficient parameter estimation for self-similar processes.", *The Annals of Statistics*, 17 (1989): 1749 – 1766.

Diebold, Francis X. and Nason, James A.. "Nonparametric exchange rate prediction?", *Journal of International Economics*, 28 (1990): 315 – 332.

Engel, Charles and Hamilton, James D.. "Long swings in the dollar: Are they in the data and do markets know it?", *American Economic Review*, 80 (September 1990): 689 – 713.

Fox, R. and Taqqu, M.S.. "Large-sample properties of parameter estimates for strongly dependent stationary Gaussian time series.", *The Annals of Statistics*, 14 (1986): 517 – 532.

Frenkel, J.A. and Mussa, M.L.. "Monetary and fiscal policies in an open economy.", *American Economic Review*, 71 (1980): 253 – 258.

Gaab, Werner. *Devisenmärkte und Wechselkurse. Eine theoretische und empirische Analyse.* Berlin, Heidelberg et al., Springer Verlag, 1983.

Granger, C.W.J. and Joyeux, R.. "An introduction to long-memory time series models and fractional differencing.", *Journal of Time Series Analysis*, 1 (1980).

Hamilton, James D.. *Time Series Analysis.* Princeton, Princeton University Press, 1994.

Hosking, J.R.M.. "Fractional differencing.", *Biometrika*, 68 (1981): 165 – 176.

Hsieh, David A.. "Modeling heteroskedasticity in daily foreign exchange rates.", *Journal of Business and Economic Statistics*, 7 (1989): 307 – 317.

Liu, Christina Y. and He, Jia. "A variance-ratio test of random walks in foreign exchange rates.", *The Journal of Finance*, 46 (1991): 773 – 785.

Lo, Andrew W.. "Long-term memory in stock market prices.", *Econometrica*, 59 (September 1991): 1279 – 1314.

Lo, Andrew W. and MacKinlay, C.. "Stock market prices do not follow random walks.", *Review of Financial Studies*, 1 (1988): 41 – 66.

Logue, Dennis E., Sweeney, Richard J., and Willett, Thomas D.. "Speculative behavior of foreign exchange rates.", *Journal of Business Research*, 6 (1978): 159 – 174.

Lucas, Robert E. Jr. "Interest rates and currency prices in a two-country world.", *Journal of Monetary Economics*, 10 (November 1982): 335 – 60.

McLeod, A.I. and Hipel, K.W.. "Preservation of the rescaled adjusted range. 1. A reassessment of the Hurst phenomenon.", *Water Resources Research*, 14 (1978): 491 – 508.

Mussa, Michael. "The exchange rate, the balance of payments, and monetary and fiscal policy under a regime of controlled floating.", *Scandinavian Journal of Economics*, 78 (1976): 229 – 248.

Robinson, Peter M.. "Time series with strong dependence.", Invited paper presented at the 6th World Congress of the Econometric Society in Barcelona, August 1990, 1990.

Schmidt, Christoph M. and Tschernig, Rolf. "The identification of fractional ARIMA models.", Discussion paper, Sonderforschungsbereich 373, Department of Economics, Humboldt University, Berlin, 1994.

Sowell, Fallaw. "Maximum likelihood estimation of stationary univariate fractionally integrated time series models.", *Journal of Econometrics*, 53 (1992): 165 – 188.

Tschernig, Rolf. *Wechselkurse, Unsicherheit und Long Memory.* Heidelberg, Physica-Verlag, 1994.

Whittle, Peter. *Hypothesis Testing in Time Series Analysis.* Uppsala, Almquist & Wiksells Boktryckeri AB, 1951.

Table 1: Estimates of ARFIMA$(2, d, 0)$ specifications for daily changes of DM/US-Dollar rates

Parameters	1973 - 90	1976 - 90	1980 - 90	1973 - 80
α_1	-0.094***	-0.081***	-0.109***	-0.049
	(0.027)	(0.030)	(0.036)	(0.040)
α_2	-0.059***	-0.021*	-0.044*	-0.089***
	(0.020)	(0.021)	(0.026)	(0.029)
d	0.079***	0.059***	0.076***	0.076***
	(0.022)	(0.025)	(0.030)	(0.033)
ξ	0.49	0.49	0.57	0.38

NOTES: standard errors are given in parenthesis. Significance levels of 10%, 5%, and 1% with respect to the null hypothesis are marked by *, **, and ***, respectively. The estimates are obtained by using the approximate Whittle estimator (27). For the full period the ARFIMA$(2, d, 0)$ specification was chosen by the AIC.

Table 2: Parameter values for temporally aggregated ARFIMA$(2, d, 0)$ processes of daily exchange rate changes

	Observation frequency			
Parameters	daily	weekly	monthly	quarterly
	changes of DM/US-Dollar rate			
α_1	-0.094	-0.001	-0.000	-0.000
α_2	-0.059	0.000	0.000	0.000
d	0.079	0.079	0.079	0.079
β_1		-0.013	0.002	0.000
β_2		-0.002	0.000	0.000
	changes of SF/US-Dollar rate			
α_1	-0.056	-0.000	-0.000	-0.000
α_2	-0.044	0.000	0.000	0.000
d	0.054	0.054	0.054	0.054
β_1		-0.010	0.001	0.000
β_2		-0.001	0.000	0.000

NOTES: the entries for weekly, monthly, and quarterly observation frequencies display the approximate ARFIMA$(2, d, 2)$ representations of the ARFIMA$(2, d, 0)$ process selected by the AIC for the full period of daily changes using the method described in appendix A.

Table 3: Estimates of ARFIMA($0, d, 0$)-GARCH(1,1) specifications for weekly changes of DM/US-Dollar rates

Parameters	1973 - 90	1976 - 90	1980 - 90	1973 - 80
d	0.063**	0.027	0.059*	0.110**
	(0.026)	(0.027)	(0.034)	(0.050)
ξ	0.195**	0.238*	1.771**	0.595**
	(0.098)	(0.127)	(0.259)	(0.112)
θ_1	0.076***	0.109***	0.130***	0.379***
	(0.017)	(0.027)	(0.048)	(0.121)
ϕ_1	0.920***	0.889***	0.629***	0.478***
	(0.889)	(0.028)	(0.114)	(0.123)
$\ln l$	-1584.809	-1315.700	-1019.380	-559.950

NOTES: standard errors are given in parenthesis. Significance levels of 10%, 5%, and 1% with respect to the null hypothesis are marked by *, **, and ***, respectively. The estimated ARFIMA($0, d, 0$)-GARCH(1,1) specification ranks 1st by the Schwarz criterion and 8th by the AIC for the full period. The estimation was carried out with the approximate time domain maximum likelihood method (29).

Table 4: Estimates of ARFIMA$(0, d, 0)$ specifications for monthly changes of DM/US-Dollar spot rates

Parameters	1973 - 90	1976 - 90	1980 - 90	1973 - 80
d	0.048	0.046	0.091	-0.083
	(0.054)	(0.056)	(0.067)	(0.101)
ξ	12.348	10.888***	11.575***	13.044***
	(1.216)	(1.181)	(1.482)	(2.024)
$\ln l$	-551.191	-444.177	-322.492	-224.358

NOTES: standard errors are given in parenthesis. Significance levels of 10%, 5%, and 1% with respect to the null hypothesis are marked by *, **, and ***, respectively. The estimated ARFIMA$(0, d, 0)$ specification ranks 1st by the Schwarz criterion and 11th by the AIC for the full period. The estimation was carried out with the approximate time domain maximum likelihood method (29).

Table 5: Estimates of ARFIMA(p, d, q) specifications for quarterly changes of DM/US-Dollar, SF/US-Dollar, DM/SF spot rates

Parameters	DM/US-Dollar	SF/US-Dollar	DM/SF
d	0.019		
	(0.102)		
β_1		0.144	0.120
		(0.145)	(0.135)
ξ	46	48	9

NOTES: standard errors are given in parenthesis. Significance levels of 10%, 5%, and 1% with respect to the null hypothesis are marked by *, **, and ***, respectively. All estimations are obtained with the Whittle estimator (24). The ARFIMA($0, d, 0$) specification for the DM/US-Dollar ranks 1st by AIC and Schwarz, the MA(1) specification for the SF/US-Dollar rate ranks 2nd by the AIC and 1st by the Schwarz criterion and the MA(1) specification for the DM/SF rate ranks 1st by both criteria.

Table 6: Out-of-sample forecasts

Exchange rate	forecasting horizon	weekly data	monthly data	quarterly data
DM/US-Dollar	6 months	-7.0%	-3.3%	+4.1%
	12 months	-3.0%	-2.1%	+0.6%
SF/US-Dollar	6 months	-4.1%	-4.2%	-1.1%
	12 months	-0.2%	-0.6%	0.0%

NOTES: This table shows the improvement or deterioration of the mean squared error (13) of an ARFIMA(p,d,q) specification relative to the mean squared error of a random walk with drift specification. For both exchange rates an ARFIMA$(0,d,0)$ specification is used. The initial predictions are based on the estimation period from January 1, 1973 until December 31, 1983. Until the predictions reach January 1, 1988, they are repeated while the estimation period is extended each time by one period. For quarterly data the predictions are carried out by means of the exact prediction method using the Innovations Algorithm of Brockwell and Davis (1991). All other predictions are computed with the approximate method ((34) and (35)).

Table 7: Estimates of ARFIMA$(2, d, 0)$ specifications for daily changes of SF/US-Dollar rates

Parameters	1973 - 90	1976 - 90	1980 - 90	1973 - 80
α_1	-0.056**	-0.060*	-0.080**	-0.002
	(0.027)	(0.030)	(0.037)	(0.043)
α_2	-0.044***	-0.030	-0.027	-0.067***
	(0.020)	(0.022)	(0.026)	(0.030)
d	0.054***	0.046*	0.064**	0.022
	(0.023)	(0.026)	(0.031)	(0.036)
ξ	0.64	0.65	0.65	0.62

NOTES: standard errors are given in parenthesis. Significance levels of 10%, 5%, and 1% with respect to the null hypothesis of $d = 0$ are marked by *, **, and ***, respectively. The estimates are obtained with the approximate Whittle estimator (27). For the full period the ARFIMA$(2, d, 0)$ specification was chosen by the AIC.

Table 8: Estimates of ARFIMA$(0, d, 0)$-GARCH(1,1) specifications for weekly changes of SF/Dollar spot rates

Parameters	1973 - 90	1976 - 90	1980 - 90	1973 - 80
d	0.049^{*}	0.030	0.045	0.117^{**}
	(0.027)	(0.029)	(0.035)	(0.048)
ξ	0.213^{*}	0.192	2.113^{***}	0.353^{**}
	(0.123)	(0.126)	(0.379)	(0.176)
θ_1	0.097^{***}	0.104^{***}	0.097^{**}	0.215^{***}
	(0.019)	(0.018)	(0.042)	(0.064)
ϕ_1	0.903^{***}	0.899^{***}	0.704^{***}	0.777^{***}
	(0.017)	(0.016)	(0.168)	(0.070)
$\ln l$	-1707.434	-1415.019	-1062.044	-634.283

NOTES: standard errors are given in parenthesis. Significance levels of 10%, 5%, and 1% with respect to the null hypothesis are marked by *, **, and ***, respectively. The estimated ARFIMA$(0, d, 0)$-GARCH(1,1) specification ranks 1st by the Schwarz criterion and by the AIC for the full period. The estimation was carried out with the approximate time domain maximum likelihood method (29).

Table 9: Estimates of ARFIMA$(0, d, 0)$ specifications for monthly changes of SF/US-Dollar spot rates

Parameters	1973 - 90	1976 - 90	1980 - 90	1973 - 80
d	0.040	0.058	0.082	-0.101
	(0.079)	(0.062)	(0.073)	(0.104)
ξ	15.203	14.345***	13.653***	16.779***
	(1.497)	(1.556)	(1.748)	(2.608)
$\ln l$	-572.619	-467.609	-322.562	-234.859

NOTES: standard errors are given in parenthesis. Significance levels of 10%, 5%, and 1% with respect to the null hypothesis are marked by *, **, and ***, respectively. The estimated ARFIMA$(0, d, 0)$ specification ranks 1st by the Schwarz criterion and 7th by the AIC for the full period. The estimation was carried out with the approximate time domain maximum likelihood method (29).

Table 10: Estimates of various ARFIMA(p, d, q) specifications for daily changes of DM/SF rates

Parameters	1973 - 90	1976 - 90	1980 - 90	1973 - 80
α_1	-0.959***		-0.712***	-0.198***
	(0.089)		(0.305)	(0.024)
α_2	-0.346***		-0.494***	-0.051**
	(0.081)		(0.206)	(0.024)
d		0.016		
		(0.013)		
β_1	-0.823***		-0.717***	
	(0.092)		(0.295)	
β_2	-0.193**		-0.549***	
	(0.086)		(0.194)	
ξ	0.18	0.13	0.08	0.34

NOTES: standard errors are given in parenthesis. Significance levels of 10%, 5%, and 1% with respect to the null hypothesis are marked by *, **, and ***, respectively. The estimates are obtained with the approximate Whittle estimator (27). For each period, the specification was chosen separately by the AIC.

Table 11: Estimates of ARFIMA$(0, d, 0)$-GARCH(1,1) specifications for weekly changes of DM/SF spot rates

Parameters	1973 - 90	1976 - 90	1980 - 90	1973 - 80
d	0.038	0.055	-0.004	0.075*
	(0.029)	(0.034)	(0.037)	(0.042)
ξ	0.176***	0.168***	0.151***	0.242***
	(0.027)	(0.025)	(0.035)	(0.042)
θ_1	0.239***	0.263***	0.212***	0.389***
	(0.050)	(0.056)	(0.057)	(0.119)
ϕ_1	0.665***	0.628***	0.686***	0.495***
	(0.071)	(0.074)	(0.087)	(0.123)
$\ln l$	-916.370	-726.307	-483.576	-427.236

NOTES: standard errors are given in parenthesis. Significance levels of 10%, 5%, and 1% with respect to the null hypothesis are marked by *, **, and ***, respectively. The estimated ARFIMA$(0, d, 0)$-GARCH(1,1) specification ranks 1st by the Schwarz criterion and 2nd by the AIC for the full period. The estimation was carried out with the approximate time domain maximum likelihood method (29).

Table 12: Estimates of AR(1)-ARCH(1) specifications for monthly changes of DM/SF spot rates

Parameters	1973 - 90	1976 - 90	1980 - 90	1973 - 80
α_1	0.084	0.131	0.151	0.031
	(0.081)	(0.079)	(0.093)	(0.104)
ξ	2.373***	1.748***	1.698***	3.855***
	(0.311)	(0.261)	(0.285)	(0.775)
θ_1	0.338***	0.425***	0.160	0.346**
	(0.117)	(0.135)	(0.123)	(0.175)
$\ln l$	-403.517	-317.669	-211.111	-182.290

NOTES: standard errors are given in parenthesis. Significance levels of 10%, 5%, and 1% with respect to the null hypothesis are marked by *, **, and ***, respectively. The estimated AR(1)-ARCH(1) specification ranks 1st by the Schwarz criterion and by the AIC for the full period. The estimation was carried out with the approximate time domain maximum likelihood method (29).