Multivariate Long-Memory ARCH Modelling for High Frequency Foreign Exchange Rates.

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February 1998

Abstract

We estimate here several trivariate FIGARCH models on three series of intra-day FX rates returns: USD/DEM, USD/GBP, and USD/JPY. We consider the trivariate constant conditional correlation CCC-FIGARCH, the unrestricted trivariate FIGARCH, and a trivariate double long-memory model combining an ARFIMA regression function with an unrestricted trivariate FIGARCH skedastic function. Estimation results show that: (*i*) the three series are anti-persistent and share a common degree of short-range dependence, (*ii*) the series USD/DEM and USD/JPY have the same regression function, (*iii*) the three series share the same degree of long-memory in their conditional variance, (*iv*) the conditional covariances Cov_t(USD/DEM, USD/JPY) and Cov_t(USD/DEM,USD/GBP) have a common degree of persistence, although this degree is different from the degree of long-memory of the conditional variances, (*v*) the unrestricted FIGARCH model dominates the CCC-FIGARCH model, although (*vi*) the seasonality in the volatility of these series cannot be captured by FIGARCH models.

Key-words: intra-day data, long-memory, anti-persistence, heteroskedasticity, multivariate models, multivariate unconstrained FIGARCH model, double long-memory, multivariate ARFIMA-FIGARCH model, second generation models.
JEL Classification: C00, C32, G00

*This paper appeared in the proceedings of the Conference "High-Frequency Data in Finance II", organised by Olsen & Associates, Zurich, 1-3 April 1998.
This paper has been largely written during my visits at GREQAM. I thank Louis-André Gérard-Varet for providing a respectable research environment, Russell Davidson for computing help, participants to the GREQAM statistical seminar, and Olsen & Associates for providing the data. The usual caveats apply.
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1 Introduction

The statistical classes of long-memory stochastic variance processes are now extensively used for the modeling of daily speculative returns, as their theoretical properties, established by Robinson and Zaffaroni (1997), Ding and Granger (1996), Harvey (1993) and Breidt, Crato and de Lima (1998), match the empirical properties of these returns: they are serially uncorrelated, but their absolute value and squares exhibit long-range dependence, and their conditional variance is predictable.

We consider here the class of long-memory ARCH processes, which extends the class of ARCH/GARCH models (Engle, 1982, Bollerslev, 1986) to fractional situations. This class has been introduced in Robinson (1991), and developed from different points of view by Ding and Granger (1995, 1996), Baillie, Bollerslev, and Mikkelsen (1996), Bollerslev and Mikkelsen (1996), McCurdy and Michaud (1996), Teyssière (1998).

In a previous work (Teyssière, 1997a), we have considered the class of long-memory ARCH processes in a multivariate framework, by modeling all the conditional second moments. This extension was motivated by the similarity of the estimated long-memory component for several series. This common long-range component in the volatility, also observed by Ray and Tsay (1997), and Henry and Payne (1997), suggests an extension to a multivariate model in which the hypothesis of a common persistent component can be tested by using the standard likelihood ratio (LR) test. A common long-range component in a set of conditional covariances is of interest for long-term forecasts since the relative variations between these covariances are only transitory. This concept of co-persistence developed by Bollerslev and Engle (1993) for the class of Integrated ARCH processes, which resembles to the idea of cointegration/fractional cointegration for conditional mean processes, is still lacking of connection with theoretical models.(See Pagan, 1996)

The most simple multivariate long-memory ARCH model is the long-memory extension of the multivariate constant conditional correlation GARCH model (CCC-GARCH), proposed by Bollerslev (1990). The CCC-GARCH model assumes that the conditional covariances are proportional to the product of the corresponding conditional standard deviations, and are characterized by a single parameter, the constant conditional correlation. This parsimonious model is of interest for high-dimensional multivariate processes. However, this assumption is very restrictive, as the multivariate CCC long-memory ARCH model amounts to a juxtaposition of univariate long-memory ARCH models. The interest of a multivariate model comes from the possibility of explicitly modeling the conditional covariances between the time series. This point is important in this new research area as there is no theoretical model explaining long-memory in the conditional second moments. We can mention some explanations such as the heterogeneous market hypothesis developed by Müller, Dacorogna, Davé, Pictet, Olsen and Ward (1993), or the combination of Clark's information arrival model and its numerous refinements, with Granger's (1980) aggregation model, as suggested by Henry and Payne (1997). An ongoing work with Kirman based on his noise traders models appears promising.

Thus, we introduced the class of unrestricted long-memory ARCH process in which we model the conditional covariances as long-memory ARCH processes. We applied this new class to the bivariate modeling of two series of daily returns on foreign exchange (FX) rates, the USD/DEM and USD/GBP FX rates, and we observed that the three elements of the bivariate covariance matrix share the same long-memory component.

This empirical result motivated this present work. As Olsen & Associates released a new dataset of financial data at intra-day frequency, we decided to apply this class of models to the 25 series of intra-day FX rates returns, for improving our knowledge of the phenomenon of long-memory in the conditional variance, and checking whether we observe similar results on intra-day FX rates.

This paper is organized as follows. We present in section **2** the concepts of long-range dependence. The class of multivariate long-memory ARCH models is introduced in section **3**. The most interesting characteristics of the data are given in section **4**, and the estimation results are discussed in section **5**. Section **6** concludes.

2 Long-range and short-range dependent time series

We can introduce the concept of long-range dependence of a time series, i.e., dependence between very distant observations, by using the class of self-similar processes. A process Y_t is self-similar if its distribution and the distribution of the rescaled process $c^H Y_{ct}$ with stretching factor c and time scale ct, are the same. The self-similarity parameter, also called the Hurst exponent, parsimoniously summarizes the degree of persistence of the series. Self-similar processes with stationary increments and finite second moments are called long-memory processes. Long-memory processes can also be characterized by their fractional degree of integration d, with $d = H - 1/2$, which allows for a more precise measure of the degree of persistence of a series than the two integer alternatives 0 and 1 of the unit root literature. If $d \in (0, 1/2)$, the autocorrelations of the process are not summable, $\sum_{k=-\infty}^{\infty} \rho(k) = \infty$, and the process has long-memory or is persistent. If $d \geq 1/2$, the process is non-stationary, but is mean reverting if $d < 1$. If $d \in (-1/2, 0)$, the autocorrelations sum up to zero, and the process has short-range dependence, or is anti-persistent.[1] Short-range dependence is rarely considered in the econometric literature, except when the series is over-differenced, but we will see in section **4** that the series of intra-day FX rates returns are anti-persistent.

The fractional degree of integration d can be estimated in several ways. Mandelbrot (1975) advocates the use of the R/S statistic as it can be applied to long-tailed α-stable

[1] See Beran (1994) and Robinson (1994) for an exhaustive presentation of long-memory processes.

processes with $\alpha \in (0, 2)$. This is of interest for applied works in finance as most financial time series are strongly leptokurtic. The semi-parametric estimators of d in the spectral domain are based on the approximation of the spectrum $f(\lambda)$ of a long-memory process near the zero frequency by $f(\lambda) = G|\lambda|^{-2d}$. Robinson (1995) established the asymptotic properties of a spectral estimator for $d \in (-1/2, 1/2)$ suggested by Künsch (1987) based on the discrete version of Whittle approximate likelihood estimator, given by:

$$\hat{d} = \arg\min_{d} \left\{ \ln\left(\frac{1}{m} \sum_{j=1}^{m} \frac{I(\lambda_j)}{\lambda_j^{-2d}} \right) - \frac{2d}{m} \sum_{j=1}^{m} \ln(\lambda_j) \right\} \tag{1}$$

where $I(\lambda)$ is the periodogram estimated in a neighborhood of the zero frequency, $\lambda_j = \pi j/n, j = 1, \ldots, m \ll n$, converging slowly to zero as the sample size increases: the bandwidth parameter tends to infinity with n but the ratio m/n tends to zero.

The parameter d can be estimated in the framework of a parametric model such as the fractional Gaussian noise, or its ARFIMA extension which generalizes the ARIMA model by replacing the difference operator by the d^{th} fractional difference operator defined as:[2]

$$(1-L)^d = \sum_{k=0}^{\infty} \alpha_k L^k, \quad \text{with} \quad \alpha_0 = 1,\, \alpha_k = \prod_{j=1}^{k} \left(1 - \frac{1+d}{j}\right) \tag{2}$$

The sequence of coefficients $\{\alpha_k\}_{k=0}^{\infty}$ have an hyperbolic rate of decay as:

$$\lim_{k \to \infty} \alpha_k = \frac{-1}{\Gamma(-q)} k^{-(1+d)} \tag{3}$$

where $\Gamma(.)$ denotes the Gamma function. The ARFIMA processes, which are characterized by slowly decaying autocorrelations, are of interest for introducing long-memory processes in the conditional variance as most ARCH/GARCH type skedastic functions can have an ARFIMA parameterization.

3 Multivariate long-memory conditional variance processes

A long-memory component can be included in the functional form of the conditional variance of a process. Define a conditional heteroskedasticity model as

$$y_t = \mu(y_t) + \varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d.}\, N(0, \sigma_t^2) \tag{4}$$

[2]See Granger (1980), Hosking (1981).

where $\mu(y_t)$ denotes the regression function, the conditional variance σ_t^2 depends on the information set Ω_t consisting of everything dated $t-1$ or earlier. Engle (1982) proposed the ARCH(p) skedastic function:

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 \tag{5}$$

where $\alpha(L)$ is a lag polynomial of order p. Robinson (1991) considered the general case of dynamic conditional heteroskedasticity by resorting to long-memory infinite order lag polynomials with rate of decay given by equation (3), and introduced the long-memory ARCH model as:

$$\sigma_t^2 = \sum_{j=1}^{\infty} \alpha_j \varepsilon_{t-j}^2 \tag{6}$$

Hence, past disturbances have a hyperbolically decaying effect on the conditional variance. Several long-memory ARCH models have been proposed in the literature.[3] With the notable exceptions of Granger and Ding (1995) and Ding and Granger (1996), all of them are based on the ARMA representation of the class of ARCH/GARCH processes. This representation allowed Engle and Bollerslev (1986) to model the observed persistence in the conditional variance of financial returns by introducing the class of IGARCH processes, which is the ARIMA parameterization of a GARCH process. Because the IGARCH process implies an infinite persistence of the shocks on the conditional variance, which does not match the observed slowly decaying persistence in the variance of financial time series data, Baillie, Bollerslev and Mikkelsen (1996) logically considered the ARFIMA parameterization and introduced the FIGARCH(p, d, q) process:

$$\phi(L)(1-L)^{d_\varepsilon}\varepsilon_t^2 = \omega + (1-\beta(L))\,\nu_t \tag{7}$$

where where $\beta(L)$ and $\phi(L)$ are lag polynomials of respectively finite order p and q, the roots of $1-\beta(L)$ and $\phi(L)$ being outside the unit circle, and $\nu_t = \varepsilon_t^2 - \sigma_t^2$ is a martingale difference. As this ARFIMA in ε_t^2 has a strictly positive drift ω, the series ε_t^2 is not covariance stationary. The parameters of this model should satisfy some conditions for insuring the conditional variance to be strictly positive.[4]

We extended the univariate long-memory ARCH models to a multivariate framework because estimation results of univariate models have shown that some time series appear to

[3] See references in the introduction.

[4] The sufficient conditions for a FIGARCH($1, d, 1$) are (Bollerslev and Mikkelsen, 1996)

$$\omega > 0,\ \beta - q \le \phi \le (2-q)/3,\ q\,(\phi - (1-q)/2) \le \beta(\phi - \beta + q)$$

The positivity conditions for higher order models are more complicate. These conditions restrict the flexibility of the FIGARCH model and its extensions.

share a common degree of long-memory in their conditional variance. Define a multivariate long-memory ARCH processes as:

$$\mathbf{Y_t} = \mu(\mathbf{Y_t}) + \varepsilon_\mathbf{t}, \quad \varepsilon_\mathbf{t} \sim \text{i.i.d.}\mathbf{N}(\mathbf{0}, \mathbf{\Sigma_t}) \tag{8}$$

where $\boldsymbol{\mu}(\boldsymbol{Y_t})$ denotes the vector regression function, $\boldsymbol{\varepsilon_t}$ is a n-dimensional vector of error terms with conditional covariance matrix $\boldsymbol{\Sigma_t}$. The constant conditional correlation (CCC) long-memory ARCH model assumes that $\boldsymbol{\Sigma_t}$ has as typical element $s_{ij,t}$, with

$$\begin{aligned} s_{ii,t} &= \sigma^2_{ii,t} = \frac{\omega_i}{1-\beta_i(1)} + \left(1 - \frac{(1-\phi_i(L))(1-L)^{d_{i,i}}}{1-\beta_i(L)}\right)\varepsilon^2_{i,t},\ i = 1, \ldots, n \\ s_{ij,t} &= \rho_{ij}\sigma_{ii,t}\sigma_{jj,t}, \quad i,j = 1, \ldots, n \quad i \neq j \end{aligned} \tag{9}$$

where $\rho_{ij} \in (-1, 1)$ denotes the constant conditional correlation. This CCC restricted model insures that the conditional variance is always positive definite, and that the model is stationary if and only if all its main diagonal elements are stationary, i.e., if $d_{i,i} \in [0,1]\,\forall i$.

However, our previous works have shown that this restriction is too strong and that the dynamic of the conditional covariances is more complex: the CCC model is dominated by the unrestricted long-memory ARCH model in which we model the conditional covariances as a long-memory process. We have proposed an unrestricted multivariate long-memory ARCH model, in which the conditional covariance matrix $\boldsymbol{\Sigma_t}$, with typical element $s_{ij,t}$, is defined as:

$$s_{ij,t} = \frac{\omega_{ij}}{1-\beta_{ij}(1)} + \left(1 - \frac{(1-\phi_{ij}(L))(1-L)^{d_{ij}}}{1-\beta_{ij}(L)}\right)\varepsilon_{i,t}\varepsilon_{j,t}, \quad i,j = 1, \ldots, n \tag{10}$$

As there is no analytical set of conditions for insuring $\boldsymbol{\Sigma_t}$ to be positive definite, we implement numerically this constraint in the estimation procedure. The estimated parameters should also insure that the multivariate long-memory ARCH process is stationary, i.e., the conditional covariance is a measurable function on the information set Ω_t and $\text{trace}(\mathbf{\Sigma_t}\mathbf{\Sigma_t}^\top)$ should be finite almost surely. This condition holds if the moments are bounded, i.e., $\text{E}(\text{trace}(\mathbf{\Sigma_t}\mathbf{\Sigma_t}^\top)^\mathbf{p})$ is finite for some p.[5] As our estimation results show that the two first moments of $\text{trace}(\hat{\mathbf{\Sigma}}_\mathbf{t}\hat{\mathbf{\Sigma}}_\mathbf{t}^\top)$ for the unrestricted model are finite and bounded by the corresponding moments of $\text{trace}(\hat{\mathbf{\Sigma}}_\mathbf{t}\hat{\mathbf{\Sigma}}_\mathbf{t}^\top)$ for the CCC model which is stationary, we can consider that the estimated process is stationary.

The drawback of this rich parameterization is a lack or parsimony: the number of parameters of the conditional covariance matrix of a n-dimensional FIGARCH$(1,d,1)$ is equal to $4n(n+1)/2$ which is large for $n \geq 4$. However, this model allows ones to test the

[5]See Bollerslev, Engle and Nelson (1994).

restriction of equality of parameters across the components of the conditional covariance matrix. Very interestingly, we have seen in this previous work on daily FX rates returns that all the components of the conditional covariance matrix, i.e., the conditional variances and the conditional covariances have the same long-memory component, i.e., $d_{ij} = d$, $\forall i, j$.

As we assume that the error terms are normally distributed, the log-likelihood function of the multivariate long-memory ARCH model is:

$$\mathcal{L}_T(\zeta) = -\frac{nT}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left(\ln|\boldsymbol{\Sigma_t}| + \boldsymbol{\varepsilon_t}^\top \boldsymbol{\Sigma_t^{-1}} \boldsymbol{\varepsilon_t}\right) \tag{11}$$

where ζ and T respectively denote the set of parameters and the sample size. The robust estimators of the variances are given by the heteroskedastic consistent covariance matrix $T^{-1}\mathcal{H}^{-1}(\hat{\zeta})\,\mathcal{I}(\hat{\zeta})\,\mathcal{H}^{-1}(\hat{\zeta})$ where $\mathcal{H}(\hat{\zeta})$ and $\mathcal{I}(\hat{\zeta})$ respectively denote the Hessian and the outer-product-of-the-gradient matrices evaluated at the Quasi Maximum Likelihood (QML) estimates $\hat{\zeta}$.

4 The data

We use here the latest release of the Olsen & Associates High Frequency dataset, HFDF96. This dataset contains 25 FX rates recorded every 30 minutes. A typical record consists of the date, the time, the offset of the previous nearest datum, the bid and ask prices associated with this previous nearest datum, the offset of the next nearest datum and the bid and ask price associated with this datum. The offset of the previous nearest datum is ≥ 0, and is equal to zero if the quote is observed at the 30 minutes interval. If no observation is available at a 30 minute interval τ, we approximate the exchange rate at this point of time by taking a linear combination of the nearest previous and next quote, the weights being proportional to the inverse of the offsets of the previous and next nearest datum.[6] As the FX market is a 24 hours global market inactive only during the week-ends, we discard the observations from Friday 10.30PM to Sunday 10.30PM, and the holidays, e.g., New Year's Day, leaving 12528 observations.

We consider here the logarithmic middle price $x(t)$ defined by

$$x(t) \equiv \frac{x_t^{bid} + x_t^{ask}}{2} \quad \text{with} \quad x_t^{bid} = \ln(p_{bid,t}),\ x_t^{ask} = \ln(p_{ask,t}), \tag{12}$$

where $p_{bid,t}$ and $p_{ask,t}$ respectively denote the bid and ask price at time t. This variable behaves symmetrically when the price is inverted. Next, we define the returns $r(t)$ as:

$$r(t) = x(t) - x(t - \Delta t) \tag{13}$$

[6]Such interpolation mechanism has been used by Andersen and Bollerslev (1996).

where Δt represents the time interval between two consecutive observations. In our case, $\Delta t = 30$ minutes. As these returns are unit-less, they can be directly compared.

In a first step, we model a univariate LM-ARCH model on the 25 series of returns. As three series, the USD/DEM, GBP/USD, and USD/JPY, appeared to share the same degree of long memory in their conditional variance, we decided to model these series with a trivariate FIGARCH process.[7] This common long-range property is not surprising as Guillaume *et al.* (1997) reported that these FX rates are the most frequently quoted, and are certainly affected by the same set of events.

Table 1: Semiparametric estimates of the degree of long-range dependence d_y of the series of returns, their absolute value and their squares, for several values of the bandwidth parameter $m = n/4, n/8, n/16$.

	$r(t)$			$\|r(t)\|$			$r(t)^2$		
FX	n/4	n/8	n/16	n/4	n/8	n/16	n/4	n/8	n/16
USD/DEM	-0.013	-0.065	-0.162	0.269	0.298	0.435	0.170	0.206	0.379
GBP/USD	-0.029	-0.036	-0.091	0.288	0.391	0.491	0.215	0.308	0.429
USD/JPY	-0.078	-0.037	-0.072	0.301	0.352	0.534	0.231	0.274	0.479

Table 1 displays the degree of fractional integration d_y of the selected series of returns their absolute value ands their squares, estimated by using Robinson's (1995) estimator. It appears that these series exhibit short-range dependence, a property already noticed by Henry and Payne (1997) on FX rates returns at 10-minute intervals. This negative serial correlation can be explained by the bid-ask bounce (Roll, 1984), or the lack of consensus among market traders on the impact of news on the direction of prices, which persistently bounce (See Guillaume *et al.*, 1997). The series of absolute returns and squared returns display a significant degree of long-memory, which confirms that these series have a long-memory component in their volatility. A second stylized fact is the existence of a strong seasonality in the volatility, caused by the inappropriateness of the calendar time to the time heterogeneity of the market activity. Mandelbrot and Taylor (1967) introduced the concept of intrinsic or fractal time of financial markets, which was further extended by Mandelbrot to the concept of multi-fractal time by assuming that the rescaling factor is a function of time:[8]

$$x(t + dt) - x(t) = dt^{H(t)} \tag{14}$$

where the Hurst exponent varies through time. This leads Dacorogna *et al.* (1993) to

[7]The other multivariate LM-ARCH, used in Teyssière (1997a), did not perform well on these data.
[8]See Mandelbrot (1997).

propose a different time scale, called ϑ-scale, in which the time is normalized by the level of market activity. The seasonality of the volatility of intra-day returns, which is more noticeable on the series of absolute returns than on the series of squared returns, casts some doubts on the relevance of the use of long-memory GARCH type skedastic functions for these data.[9] As we are rather concerned with the long-memory properties of the series, we can consider that these seasonal local variations will not interfere too much with the long term variations. This can be verified in a further work by considering variables in ϑ-time.

5 Estimation results

We consider in a first approach to model the negative serial correlation of the conditional mean by an autoregressive regression function. We select the order of the model by using the Bayes Information Criterion, although the properties of the BIC have not been formally established in the presence of conditional heteroskedasticity.[10]

The specification for the conditional mean vector $\boldsymbol{Y_t} = [y_{1,t}, y_{2,t}, y_{3,t}]^\top$, where $y_{1,t}$, $y_{2,t}$ and $y_{3,t}$ respectively denote the series of 1000×USD/JPY, 1000×USD/GBP, and 1000× USD/DEM returns, is

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} + \begin{pmatrix} \psi_{1,1} y_{1,t-1} & + & \psi_{1,2} y_{1,t-2} \\ \psi_{2,1} y_{2,t-1} & + & \psi_{2,2} y_{2,t-2} \\ \psi_{3,1} y_{3,t-1} & & \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{pmatrix} \tag{15}$$

$$\text{with} \quad \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{pmatrix} \sim \text{i.i.d.} N \left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} s_{11,t} & s_{12,t} & s_{13,t} \\ s_{21,t} & s_{22,t} & s_{23,t} \\ s_{31,t} & s_{32,t} & s_{33,t} \end{pmatrix} \right] \tag{16}$$

[9] See Guillaume *et al.* (1995). Seasonal GARCH have been proposed by Bollerslev and Ghysels (1994). However, the seasonality pattern of intra-day data requires more sophisticated adjustment techniques.

[10] The presence of a long-memory component in the conditional variance is likely to affect this criteria, as Beran and Bhansali (1997) have shown that the presence of long-memory in the conditional mean implies a modification of the BIC so as to obtain a consistent estimator of the order of the autoregressive polynomial.

The retained specification for the CCC-FIGARCH is:

$$\begin{pmatrix} s_{11,t} \\ s_{22,t} \\ s_{33,t} \\ s_{12,t} \\ s_{13,t} \\ s_{23,t} \end{pmatrix} = \begin{pmatrix} \omega_{1,1} + \left(1-(1-L)^{d_{1,1}}\right)\varepsilon_{1,t}^2 \\ \dfrac{\omega_{2,2}}{1-\beta_2(1)} + \left(1 - \dfrac{(1-\phi_2 L)(1-L)^{d_{2,2}}}{1-\beta_2 L}\right)\varepsilon_{2,t}^2 \\ \dfrac{\omega_{3,3}}{1-\beta_3(1)} + \left(1 - \dfrac{(1-\phi_3 L)(1-L)^{d_{3,3}}}{1-\beta_3 L}\right)\varepsilon_{3,t}^2 \\ \rho\sqrt{s_{11,t}}\sqrt{s_{22,t}} \\ \rho\sqrt{s_{11,t}}\sqrt{s_{33,t}} \\ \rho\sqrt{s_{22,t}}\sqrt{s_{33,t}} \end{pmatrix} \tag{17}$$

Table 2 page 15, displays the estimation results of the trivariate CCC-FIGARCH. It clearly appears that the three series share the same degree of long-memory in their conditional variance, $d = 0.17$. Furthermore, the series $y_{2,t}$ and s $y_{3,t}$, i.e., the USD/GBP and USD/DEM, share the same skedastic structure as the log-likelihood of the restricted model, i.e., the model with the restrictions $d_{1,1} = d_{2,2} = d_{3,3}$, $\beta_2 = \beta_3$, and $\phi_2 = \phi_3$, does not significantly differ from the log-likelihood function of the unrestricted model. Thus, the LR test does accept this restriction. This result is similar to the one observed with the same series at daily frequency. We also observe that the estimated conditional correlations are rather low.

We now compare these results with the estimation results of the trivariate unrestricted FIGARCH model. Equation (18) displays the retained specification of the conditional covariance matrix, the estimation results are given in table 3, page 16.

$$\begin{pmatrix} s_{11,t} \\ s_{22,t} \\ s_{33,t} \\ s_{12,t} \\ s_{13,t} \\ s_{23,t} \end{pmatrix} = \begin{pmatrix} \omega_{1,1} + \left(1-(1-L)^{d_{1,1}}\right)\varepsilon_{1,t}^2 \\ \dfrac{\omega_{2,2}}{1-\beta_2(1)} + \left(1 - \dfrac{(1-\phi_2 L)(1-L)^{d_{2,2}}}{1-\beta_2 L}\right)\varepsilon_{2,t}^2 \\ \omega_{3,3} + \left(1-(1-\phi_3 L)(1-L)^{d_{3,3}}\right)\varepsilon_{3,t}^2 \\ \omega_{1,2} + \left(1-(1-L)^{d_{1,2}}\right)\varepsilon_{1,t}\varepsilon_{2,t} \\ \omega_{1,3} + \left(1-(1-L)^{d_{1,3}}\right)\varepsilon_{1,t}\varepsilon_{3,t} \\ \omega_{2,3} + \left(1-(1-L)^{d_{2,3}}\right)\varepsilon_{2,t}\varepsilon_{3,t} \end{pmatrix} \tag{18}$$

The unrestricted trivariate FIGARCH model dominates the restricted CCC-FIGARCH, and since the two first moments of trace($\hat{\mathbf{\Sigma}}_\mathbf{t}\hat{\mathbf{\Sigma}}_\mathbf{t}^\top$) for the unrestricted trivariate model are bounded, the estimated model appears stationary. The three conditional variances still have the same degree of long-memory. However, the USD/DEM and USD/GBP do no longer have a common skedastic structure. Interestingly, the three conditional covariances do not share a common degree of persistence with the conditional variances. The conditional covariances Cov_t(USD/DEM,USD/JPY) and Cov_t(USD/DEM,USD/GBP) share the same degree of persistence, while the degree of persistence of Cov_t(USD/JPY,USD/GBP) is rather low. These estimation results are consistent with the estimated results of the conditional correlations of the trivariate CCC-FIGARCH model: the correlation coefficients ρ_{12} between USD/DEM and USD/JPY, and ρ_{23} between USD/GBP and USD/DEM where close, although the restriction $\rho_{12} = \rho_{23}$ is rejected by the LR test.

We also observe that the coefficient ϕ and β of the three conditional covariances are not statistically significant. This may be caused by the low conditional correlation between these series, or a misspecification of the FIGARCH model, which cannot capture very rich dynamics, although the positivity conditions are not required for the conditional covariance process.

The inadequacy of the FIGARCH model to capture the seasonality of the volatility is illustrated by the Ljung-Box statistic $Q^2(k)$ based on squared standardized residuals $\hat{\varepsilon}_t^2/\hat{\sigma}_t^2$. Under the null hypothesis of no remaining heteroskedasticity at the order k, this statistic is χ^2_{k-l} distributed where l is the number of GARCH parameters of the skedastic function. Although the distribution of this statistic in the multivariate case has not been established, we observe that the null hypothesis is accepted for the series USD/JPY for all orders. However, this statistic is rejected for the two other series, when the order is over 48, i.e., one day of observations. For the statistic $Q^A(k)$, based on the absolute standardized residuals, $|\hat{\varepsilon}_t/\hat{\sigma}_t|$, the null hypothesis is rejected for all orders. This advocates the use of deasonalized data in ϑ-time.

We extend this trivariate model by considering a fractionally integrated ARMA (ARFIMA) regression function. This multivariate double long-memory model extends the univariate models developed in Teyssière (1997b) for short-range dependent daily returns with long-range dependence in their conditional variance. Since these series of intra-daily returns are anti-persistent, we included this short-range dependent component in the specification of the regression function. The LR test accepts the hypothesis that the component is the same for the three series. Furthermore, the series $y_{1,t}$ and $y_{3,t}$, i.e., the USD/JPY and USD/DEM series, also share the same regression function. The retained specification for the conditional mean vector is:

$$\begin{pmatrix} (1-\psi_{1,1}L)(1-L)^{d_y}(y_{1,t}-\mu_1) \\ (1-\psi_{2,1}L)(1-L)^{d_y}(y_{2,t}-\mu_2) \\ (1-\psi_{1,1}L)(1-L)^{d_y}(y_{3,t}-\mu_3) \end{pmatrix} = \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{pmatrix} \tag{19}$$

Estimation results of the restricted model are given in table 4 page 17. This model clearly dominates all the previous ones, although the problem of seasonal heteroskedasticity remains.

6 Conclusion

We have estimated here a trivariate Fractional Autoregressive FIGARCH, FAR-FIGARCH, model on three series of intra-day FX rates returns: USD/DEM, USD/GBP, and USD/JPY. Estimation results have shown that: (*i*) the three series are anti-persistent and share the same degree of anti-persistence d_y = -0.0618, (*ii*) the series USD/DEM and USD/JPY share the same regression function, (*iii*) the three series share the same degree of long-memory in their conditional variance $d = 0.1614$, and (*iv*) the conditional covariances Cov_t(USD/DEM,USD/JPY) and Cov_t(USD/DEM,USD/GBP) share the same degree of persistence $d = 0.1038$. These long-term and non-periodic similarities are observed only on the most frequently quoted FX rates. This may reflect a consensus among market traders on the long term evolution of these currencies.

These results also show the limitations of both the multivariate CCC-FIGARCH, and the standard long-memory ARCH skedastic functions which cannot model the seasonality of the volatility when measured in calendar time. The use of series in ϑ-time will allow us to go farther and consider higher-dimensional multivariate long-memory ARCH processes.

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Table 2: Estimation results of the trivariate CCC-FIGARCH. Robust standard error are between parentheses.

Parameters	Unrestricted model	Restricted model
μ_1	0.0180 (0.0066)	0.0178 (0.0066)
μ_2	-0.0057 (0.0050)	-0.0058 (0.0051)
μ_3	0.0051 (0.0057)	0.0048 (0.0057)
$\psi_{1,1}$	-0.1614 (0.0103)	-0.1613 (0.0103)
$\psi_{1,2}$	-0.0453 (0.0092)	-0.0452 (0.0092)
$\psi_{2,1}$	-0.2236 (0.0117)	-0.2234 (0.0117)
$\psi_{2,2}$	-0.0622 (0.0103)	-0.0622 (0.0102)
$\psi_{3,1}$	-0.1539 (0.0099)	-0.1539 (0.0099)
$\omega_{1,1}$	0.1750 (0.0205)	0.1766 (0.0187)
$\omega_{2,2}$	0.0476 (0.0174)	0.0464 (0.0057)
$\omega_{3,3}$	0.0548 (0.0092)	0.0525 (0.0059)
β_2	0.5261 (0.1746)	0.5471 (0.0169)
ϕ_2	0.5922 (0.1854)	0.6095 (0.0046)
β_3	0.5496 (0.0253)	β_2
ϕ_3	0.6146 (0.0095)	ϕ_2
$d_{1,1}$	0.1733 (0.0171)	0.1714 (0.0137)
$d_{2,2}$	0.1792 (0.0301)	$d_{1,1}$
$d_{3,3}$	0.1563 (0.0286)	$d_{1,1}$
ρ_{12}	0.2107 (0.0116)	0.2112 (0.0116)
ρ_{13}	0.4580 (0.0102)	0.4585 (0.0102)
ρ_{23}	0.3945 (0.0124)	0.3947 (0.0125)
Log-likelihood	-34942.601	-34944.015

Moments of trace($\hat{\boldsymbol{\Sigma}}_{\mathbf{t}}\hat{\boldsymbol{\Sigma}}_{\mathbf{t}}^{\top}$)
E(trace($\hat{\boldsymbol{\Sigma}}_{\mathbf{t}}\hat{\boldsymbol{\Sigma}}_{\mathbf{t}}^{\top}$))= 1.8829
E(trace($\hat{\boldsymbol{\Sigma}}_{\mathbf{t}}\hat{\boldsymbol{\Sigma}}_{\mathbf{t}}^{\top})^{\mathbf{2}}$)= 95.6630

Table 3: Estimation results of the trivariate unconstrained FIGARCH model. Robust standard error are between parentheses.

Parameters	Unrestricted model	Restricted model
μ_1	0.0175 (0.0064)	0.0175 (0.0064)
μ_2	-0.0058 (0.0049)	-0.0057 (0.0050)
μ_3	0.0034 (0.0056)	0.0034 (0.0056)
$\psi_{1,1}$	-0.1655 (0.0101)	-0.1654 (0.0101)
$\psi_{1,2}$	-0.0430 (0.0089)	-0.0431 (0.0089)
$\psi_{2,1}$	-0.2287 (0.0113)	-0.2291 (0.0112)
$\psi_{2,2}$	-0.0606 (0.0101)	-0.0606 (0.0101)
$\psi_{3,1}$	-0.1571 (0.0098)	-0.1572 (0.0098)
$\omega_{1,1}$	0.1822 (0.0199)	0.1842 (0.0179)
$\omega_{2,2}$	0.0567 (0.0151)	0.0559 (0.0147)
$\omega_{3,3}$	0.1231 (0.0142)	0.1232 (0.0117)
$\omega_{1,2}$	0.0586 (0.0062)	0.0586 (0.0062)
$\omega_{1,3}$	0.0922 (0.0084)	0.0943 (0.0081)
$\omega_{2,3}$	0.0684 (0.0074)	0.0665 (0.0066)
β_2	0.4919 (0.1312)	0.4951 (0.1236)
ϕ_2	0.5549 (0.1412)	0.5569 (0.1272)
ϕ_3	0.0523 (0.0205)	0.0515 (0.0194)
$d_{1,1}$	0.1656 (0.0152)	0.1625 (0.0113)
$d_{2,2}$	0.1593 (0.0252)	$d_{1,1}$
$d_{3,3}$	0.1629 (0.0196)	$d_{1,1}$
$d_{1,2}$	0.0590 (0.0083)	0.0593 (0.0082)
$d_{1,3}$	0.1073 (0.0081)	0.1030 (0.0065)
$d_{2,3}$	0.0975 (0.0109)	$d_{1,3}$
Log-likelihood	-34798.568	-34799.302

Moments of trace($\hat{\mathbf{\Sigma}}_\mathbf{t}\hat{\mathbf{\Sigma}}_\mathbf{t}^\top$)
E(trace($\hat{\mathbf{\Sigma}}_\mathbf{t}\hat{\mathbf{\Sigma}}_\mathbf{t}^\top$))= 1.7496
E(trace($\hat{\mathbf{\Sigma}}_\mathbf{t}\hat{\mathbf{\Sigma}}_\mathbf{t}^\top)^\mathbf{2}$)= 76.6262

Table 4: Estimation results of the restricted trivariate double long-memory model. Robust standard error are between parentheses.

Parameters	Restricted model
μ_1	0.0139 (0.0037)
μ_2	-0.0051 (0.0027)
μ_3	0.0041 (0.0032)
$\psi_{1,1}$	-0.1036 (0.0100)
$\psi_{1,2}$	-0.1685 (0.0118)
$\psi_{1,3}$	$\psi_{1,1}$
d_y	-0.0618 (0.0072)
$\omega_{1,1}$	0.1835 (0.0180)
$\omega_{2,2}$	0.0562 (0.0137)
$\omega_{3,3}$	0.1223 (0.0117)
$\omega_{1,2}$	0.0580 (0.0062)
$\omega_{1,3}$	0.0935 (0.0081)
$\omega_{2,3}$	0.0657 (0.0066)
β_2	0.4908 (0.1139)
ϕ_2	0.5555 (0.1173)
ϕ_3	0.0529 (0.0198)
$d_{1,1}$	0.1626 (0.0114)
$d_{2,2}$	$d_{1,1}$
$d_{3,3}$	$d_{1,1}$
$d_{1,2}$	0.0600 (0.0082)
$d_{1,3}$	0.1038 (0.0065)
$d_{2,3}$	$d_{1,3}$
Log-likelihood	-34772.184

Moments of trace($\hat{\mathbf{\Sigma}}_\mathbf{t}\hat{\mathbf{\Sigma}}_\mathbf{t}^\top$)
E(trace($\hat{\mathbf{\Sigma}}_\mathbf{t}\hat{\mathbf{\Sigma}}_\mathbf{t}^\top$))= 1.7694
E(trace($\hat{\mathbf{\Sigma}}_\mathbf{t}\hat{\mathbf{\Sigma}}_\mathbf{t}^\top)^\mathbf{2}$)= 83.3956