Long Memory in Rupee-Dollar Exchange Rate – An Empirical Study

Abstract

The Exchange rate policy of the Government has attempted to make the Indian Rupee more market driven though earlier at time the central intervened in the market to maintain stability of the exchange rate. This study uses last 13 years data to find out if there exists any long memory process in the INR-US$ Exchange Rate. The Classical R/S analysis as well as Variance Ratio Tests have been conducted in the data to figure out the results. The stationarity condition have been tested using ADF and Phillip - Perron tests and it has been observed that the return series is stationary and has no evidence of ARCH using White's ARCH test. The normality tests on the daily exchange rate returns for the last one-decade or so indicate the need to explore the application of non-linear modeling techniques while understanding exchange rate behaviour. But we come to see that the results from the persistence tests are split. The variance test clearly implies that there exists only short-term memory in the market returns as given by study above and the pattern is also not clearly established. However, the R/S analysis does give indications of long-term memory but with noise. In either case, analysis shows that the movement of exchange rate does not follow a random movement.

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Introduction

India, in recent years, has gone through many changes in financial system. The perennial BOP problem faced by the countries till 1991 forced the country to move to the path of liberalization by adopting deregulation of interest as well as exchange rate. From a fixed exchange rate mechanism when the central bank of India, RBI, played the dominant role in determining the value of the domestic currency vis-à-vis other hard currencies, specifically its exchange rate with US $ to LERMS (Liberalized Exchange Rate Management System) to a unified exchange rate system from March 1, 1993, we have moved a long and tedious path. The introduction of current account convertibility and move towards introduction of capital account convertibility has helped the Rupee-Dollar exchange rate to attain more flexibility as required by the market conditions. Over the years, there have been few instances of RBI intervention in the foreign exchange market and the system has helped the central bank to build up a very comfortable reserves position. There have many policy changes during the period and the most significant one being the replacement of draconian FERA by much user friendly FEMA. The opening up of the economy helped to attract huge inflows of foreign capital, both FDI and FII and this helped the exchange rate to stabilize. The Asian crisis of 1997-98 did not have a lingering impact on Rupee-Dollar exchange rate. The adaptation of the flexible exchange rate regime in the nineties and the accelerated integration of financial markets with globalization during 1990s made the behavior of exchange rates important to understand financial aggregates in India.

The purpose of this paper is to assess the long memory component of exchange rate for a relatively moderate open economy like India. The exchange rate fluctuations introduce a risk on return of an asset in foreign currency, and foreign investors might want to be compensated with higher risk premium. And in economies like India where such fluctuations are relatively more compared to other developed economies, foreign investors would like to have a higher return from their investments in Indian markets. This study tries to test and find if the data can be analysed in a meaningful way to develop trading and investment strategies by market participants. The study concentrates on last one decade's data as India introduced market related reforms from 1990-91 and the period of study coincides with the reforms period.

Definitions and Objectives

Time series analysis is an integral part of financial analysis. The topic is interesting and useful, with applications to the prediction of interest rates, foreign currency risk, stock market volatility, and the like. Forecasting a time series is a common problem in many domain of science, and has been addressed for a long time by statisticians and econometricians. Predictions of a financial series like stock market prices or index values or exchange rate remain a very specific task. Long term dependence of a time series describes the correlation structure of the series at long lags. If a time series exhibits long memory or the biased random walk, there is persistent temporal dependence even between observations distanced by long time lags. Such series are characterized by distinct but non-periodic cyclical patterns. The presence of long memory dynamics in asset prices would provide evidence against weak form of EMH as it implies nonlinear dependence in the first moment of the distribution and hence a potentially predictable component in the series dynamics.

The study of asset prices has begun since a long time with Fama (1965) clearly put to light the highly stochastic nature of their behaviour. L Bachelier (1914) was the first to propose the theory of random walk to characterize the change of security prices through time. Fama (1965) analyzed the distribution of a large data set. He showed that empirical evidence seems to confirm the random walk hypothesis: a series of price changes has no memory ("the past can not be used to predict the future in any meaningful way"). The main theoretical explanation that lies behind this observation is the efficient market hypothesis. It was not until 1980's that researchers began to apply the rescaled range analysis, one of the tools in the long memory theory, to financial markets and macroeconomic prices. According to the Efficient Markets Hypothesis (EMH) an efficient capital market is one in which security prices adjust rapidly to the arrival of new information, and therefore, the current prices of securities reflect all information about the security. Three sets of assumptions imply an efficient capital market: (a) an efficient market requires that a large number of competing profit-maximizing participants analyze and value securities, each independently of the others, (b) new information regarding securities come to the market in a random fashion, and the timing of one announcement is generally independent of others, and (c) the competing investors attempt to adjust security prices rapidly to reflect the effect of new information. Although the price adjustment may be imperfect, it is unbiased. This means that sometimes the market will over-adjust or under-adjust, but an investor cannot predict which will occur at any given time. Hence if a statistically significant serial dependence exists within time series of financial security prices, the community of financial analysts will immediately exploit the same. Financial assets' price changes can therefore be only explained by the arrival of new information, which, by definition can not be forecasted.

One of the key observations explained by Peters (1994) is the fact that most financial markets have a long memory; what happens today affects the future forever. In other words, current data is correlated with all past data to varying degrees. This long memory component of the market can not be adequately explained by systems that work with short memory parameters. The short memory property describes the low-orderr correlation structure of a series and for short memory, correlations among observations at long lags become negligible. Long memory systems on the other hand are characterized by their ability to remember events in the long history of time series data and their ability to make decisions on the basis of such memories. Long memomry implies the perfect arbitrage is impossible (Mandelbrot (1971)) and invalidates standard derivative pricing models based on Brownian notion and martingale assumptions.

Motivation:

Exchange Rate affects all countries small or big. Voluminous literature has been created on purchasing power parity. Managing exchange rate has become a daunting exercise for central banks. There has been currency crises that has wiped out economic wealth for many. For many countries, the actual benefits of currency depreciation are less impressive than what conventional wisdom predicts in terms of boosting export competitiveness to raising aggregate economic output. Indian exports have not substantially increased even though we had devalued our currency on certain occasions. Though the views differ as to whether, how and to what extent it might be desirable to promote competitive depreciation to suit domestic economic interests, large depreciations have also the potential to increase credit risk and the burden of debt denominated in foreign currencies. Downward pressures on exchange rates and downturns in market sentiment can be mutually reinforcing and result in higher uncovered exchange rate exposure and financial disruption.

Large exchange rate fluctuations in an environment of increased international capital mobility affect the level of inflation predictability and the pricing of financial assets. Unexpected fluctuations in inflation targets and expectations can exert downward pressures on financial market valuation as drifts from the purchasing power parity rates have the potential to generate increased uncertainty on firms' cash flows and affect their market value. Broadly defined, exposure to foreign exchange risk measures the sensitivity of the firm value, or the present value of expected future cash flows, to currency gyrations. The negative exposure of import-oriented firms to depreciation has the potential to decrease stock prices and increase the required risk premium. The effect is asymmetric with respect to import-oriented firms but even purely domestic firms may still suffer from currency depreciation because of sustainable falls in aggregate domestic demand. The aggregate impact of exchange rate variations on stock market valuation is ultimately function of the trade imbalance within the economy.

During last one decade, the Indian financial system has been subjected to substantial reforms with far reaching consequences. The reforms process centered around interest rate as well as exchange rate deregulations. These reforms process has helped in dramatic improvement in transparency level in financial markets including foreign exchange market. Before liberalization, India has only one official rate that used to be determined by the central bank of the country and market participants had very little role in the market with regard to determination of the exchange rate. As per the new economic policy 1991, exchange rate liberalization received maximum attention of the policy makers. In early 1990's, the experiments with Liberalized Exchange Rate Management System (LERMS) proved successful and slowly from dual exchange rate system the country moved to an unified exchange rate system. By 1994, the exchange rate was by and large convertible on current account and partially on capital account. Full convertibility of the exchange rate, though not in place today, is on the agenda of the central banks and to show seriousness to the issue, RBI appointed a committee to suggest roadmap for introduction of full convertibility. The country experienced the mild contagion effect of financial crisis in International markets and successfully sailed through the period of Asian crisis not significantly jeopardizing the interest of the domestic economy and fall in Indian Rupee was not high compared to other emerging Asian economies. Today the exchange rate is determined by the market forces of demand and supply and market participants play a dominant role in the determination of exchange rate. Dirty float has been replaced by a free float so far the market is concerned, though at times the central bank has to cool the excess volatility in the market with indirect intervention like issuing policy statements. Over the years more flexibility has been provided by the central banks to market participants including banks and institutions to operate in the foreign exchange market. Foreign Exchange Regulation Act has been replaced by a more friendly Foreign Exchange Management Act. Risk management system has been changing in keeping pace with change in scenario. Other reforms in the form of deregulation of interest rate, tax reforms, banking sector reforms, reforms in the external sector, etc. has also helped market participants to value assets according to their intrinsic values. Liquidity has greatly increased as the market with increase in depth of the market. International investors' access to the domestic market has also helped in increasing liquidity. All these helped in better dissemination of information and hence increased the level of efficiency in asset prices. Over the period, without adding much to the stock of external debt, there has been a quantum jump in forex reserves. This position needs to be contrasted with the 1980s, when external debt, especially short-term debt, mounted while the forex reserves got depleted. In fact, it is often held that, between 1956 and 1992, India faced balance of payments constraints in all but six years, while during the last ten years, there has never been a feeling of constraint on this account, eventhough the period coincided with liberalization of external account, global currency crisis and domestic political uncertainties.

A decade has passed since India started the liberalization process. During this period the foreign exchange market has undergone substantial change and has been subject to few shocks. As the time has passed, the direct intervention by the central bank in the foreign exchange market has come down drastically and today the central bank only issues policy guidelines to give direction to the market. The huge build of reserves of nearly US$55 billion has helped RBI to liberalise the foreign exchange inflows as well as outflow. If we look at the literature available on Indian market, very little work has been done to study the existence of long memory in foreign exchange market in India. In this study I try to investigate the presence of fractional dynamics in the exchange rate in terms of US $ (Indian Rupee / US $)

returns series. Using a long time price series is an important attribute of the present study. Foreign exchange market is very vulnerable and sensitive to exogenous shocks. Therefore, there exists a tendency that many unexplained price spikes are attributed to exogenous shocks and are kept out of modeling practices. If the data represent a long time period, such generating processes have more chances to repeat themselves in one way or another; there are more chances for modeling practices to succeed.

The potential presence of stochastic long memory in financial asset returns has been an important subject of both theoretical and empirical research. If assets display long memory, or long-term dependence, they exhibit significant autocorrelation between observations widely separated in time. Since the series realizations are not independent over time, realizations from the remote past can help predict future returns. Persistence in returns, may it be stocks or currency, has a special claim on the attention of investors because any predictable trend in returns should be readily exploitable by an appropriate strategy by the market participants.

A great deal of research work has been undertaken by experts on financial markets and economists on both Random Walk and Efficient Market Hypothesis. Literature available depicts studies concerning developed and emerging markets. Some important work has been done by Thomas (1995), Basu & Morey (1998), Nath (2001). Nath & Reddy (2002), covering Indian stock market returns but very little work has been done to study the long memory of exchange rate in India. Basu & Morey (1998) used Variance Ratio Test methodology devised by Lo and MacKinlay to test the long memory of the Indian stock market while Nath (2001) used Variance Ratio and R/S analysis to test the stock market data for long memory components. This paper makes an attempt to study Indian market with respect to its long-memory using Rupee/$ exchange rate returns during last one decade or so during which the foreign exchange market has gone through substantial liberalization process and also on few occasions it has been subject to extreme volatile situations for many reasons including the Asian currency crisis. The study attempts to examine the efficient market hypothesis on Indian conditions by implementing two important techniques that are robust to time varying volatility. The study has been based on the idea that variance keeps changing over time and hence a test like Variance Ratio test would not only help to test the random walk theory in stock prices but also being robust to time-varying volatility.

Literature Review:

A number of empirical studies (e.g. Booth, Kaen and Koveos, 1982; Cheung, 1993; Batten and Ellis 1996) employ the rescaled range statistical procedure, originally developed by Hurst (1951), to identify long-term return anomalies in currency markets. However, Fama (1998) argues that this type of anomaly may be sensitive to the method employed and will tend to disappear when alternative approaches are used. That is, they are “methodological illusions” (ibid. p.385). Given the scale of the spot Re/USD trading and the high level of information efficiency in foreign exchange markets one may be predisposed to favour the Fama (1998) view. One approach to the problem is to apply similar statistical methods but determine whether the return anomaly persists over different sample periods, or is specific to one or more subperiods. My approach to this problem differs from other researchers in that I deduce from the data the sub-periods for investigation.

Many authors have tested for long memory in asset returns, including both stocks and exchange rate returns. Lo (1991) proposed robustifying the rescaled range statistics of Hurst (1951) against short run dependence and tested for US stock indices. Jacobsen (1996) tested for long memory in US, Japanese and some West European stock index returns. Cheung (1993a) tested for long memory in exchange rate returns, while Hiemstra and Jones (1997) considered long memory in US individual stock returns. Crato (1994) and Cheung and Lai (1995) both tested for long memory in stock return indices for a number of developed markets. Greene and Fielitz and Aydogan and Booth (1988) both tested for long memory using rescaled range statistics. Barkoulas and Baum (1997) tested long memory of stock returns and Euroyen Deposit Rates. Booth, Kaen, Koveos (1982), Cheung (1993a) Cheung and Lai (1993) Bhar (1994), Feng, Lai and Lai (1994) Barkoulas, Libys and Onochie (1997a) tested long memory for spot and futures currency rates. Other authors have also tested long memory for gold prices, international spot commodity prices and commodity and stock index futures. Wei and Leuthold (2000) tested existence of long memory in agricultural futures prices.

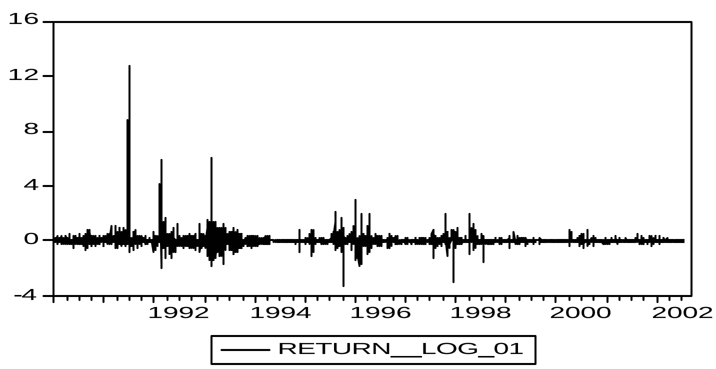
Lo (1991) developed a modified R/S method, which addresses some of the drawbacks of the classical R/S method. Using the variant of R/S analysis, Lo finds no evidence to support the presence of long memory in U.S. stock returns. Applying Lo test, which does not rely on standard regression techniques and is robust to short-term dependence, provides statistical support for the hypothesis that stock market returns follow a random walk (Ambrose, 1993). Using both the modified R/S method and the spectral regression method, Cheung and Lai (1995) find no evidence of persistence in several international stock returns series. Crato (1994) reports similar evidence for the stock returns series of the G-7 countries using exact maximum likelihood estimation.

Fung and Lo's (1993) long memory study analyzed the prices of two interest rate futures markets, Eurodollars and T-bills. The result from the classical R/S analysis and Lo's (1991) modified R/S analysis provide no evidence of the existence of long memory and support for the weak form efficient market hypothesis. Fung et al (1994) examined long memory in stock index futures by using variance ratio, R/S and auroregressive fractally integrated moving average models. All three types of analyses concluded that no long memory exists in the data. Similar tests have been pursued by many academicians but the results are mixed, but all authors agreed the identification of long memory is very important and significant in two senses: (a) the time span and strength of long memory will be an important input for investment decisions regarding investment horizons and composition of portfolios; and (b) prediction of price movements will be improved. It is also noticeable that research methodologies have developed very fast. In the 1980's the classical R/S analysis was the major tool but in 1990's the methods are being diversified with the modified R/S analysis and the AFIMA model as new techniques.

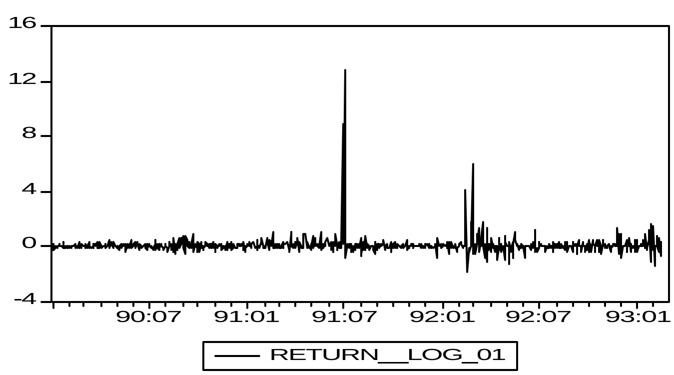
Data and Data Characteristics:

The procedures for collecting and transforming data affect any serious statistical modeling. Before initiating sophisticated statistical analysis, it is important to analyze the basic properties of the data with simple methods. Therefore, this section first presents the data used in the study, and then discusses normality, stationarity and the structure of autocorrelations and partial autocorrelations of the data. The importance of normality and stationarity is shared by many empirical studies, while the structure of autocorrelations has special significance in non-linear dynamic modeling. The INR/USD exchange rate data has been collected from the market sources that are basically mid rates and reference rates disseminated by Reserve Bank of India. The time period covers from January 1990 to November 2002. The period covers a phase when RBI has devalued the currency directly through administrative orders prior to March 1993 and after March 1993 on a few occasions with structural adjustments engineered by the central bank. The same can be viewed from the spikes (large losses/gains) in the returns data. However, the major spike in 1997-98 is due to Asian crisis that hit most of the emerging countries currencies but for India the effect was not very high. To understand the how the market efficiency improved over the years, the entire period has been divided into 11 buckets. It has been noticed that as we moved toward the later part of the time bracket, the skewness and kurtosis has significantly improved indicating that the large incidence of the data falls into a close range. The figures indicate the same.

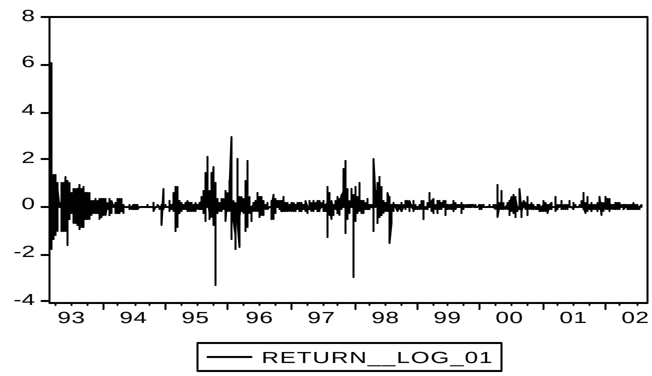
1. Returns for the period from January 1990 to November 2002



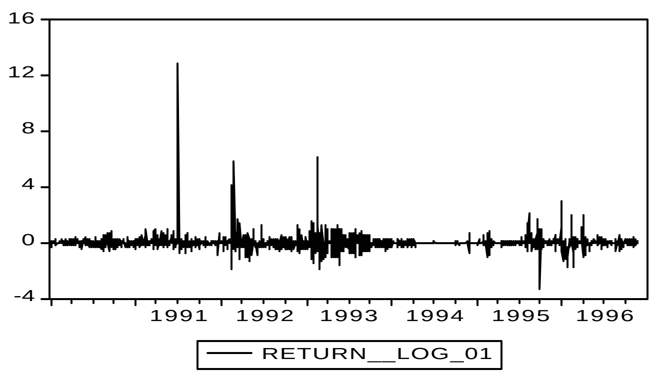
2. Returns for the period from January 1990 to February 1993



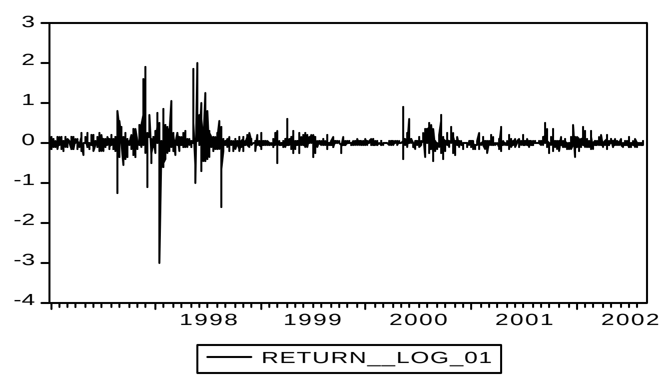
3. Returns for the period from March 1993 to November2002



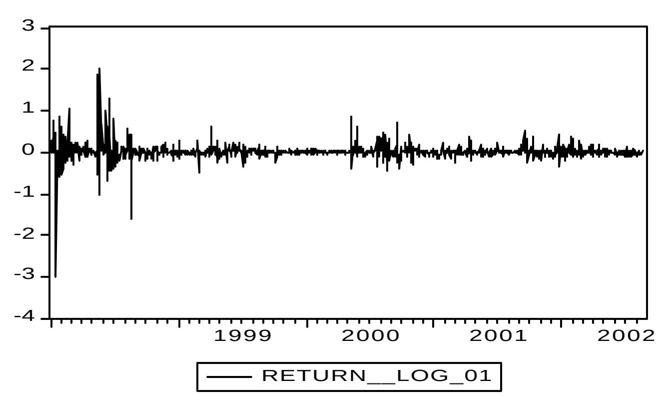
4. Returns for the period from January 1990 to December 1996



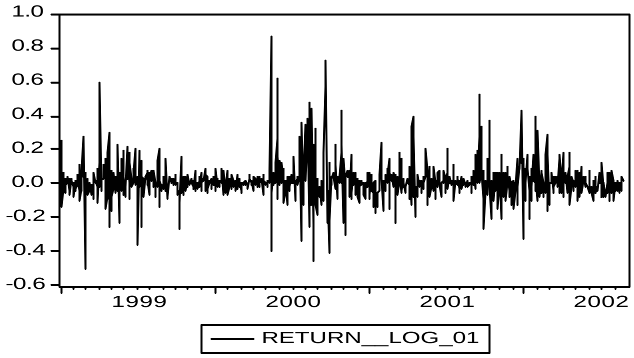
5. Returns for the period from January 1997 to November2002



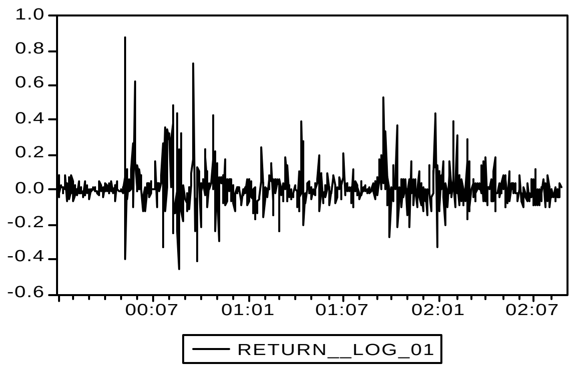
6. Returns for the period from January 1998 to November2002



7. Returns for the period from January 1999 to November2002

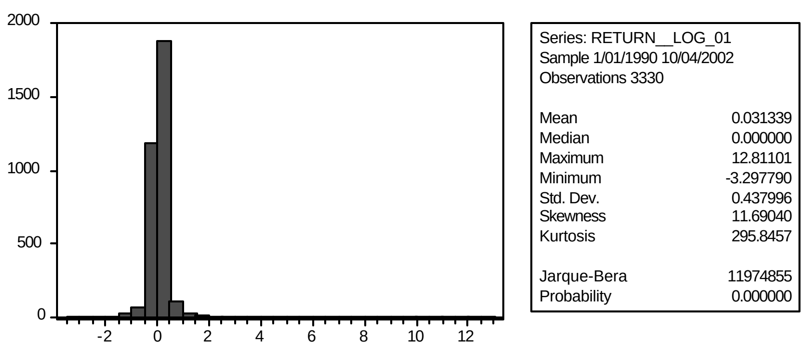


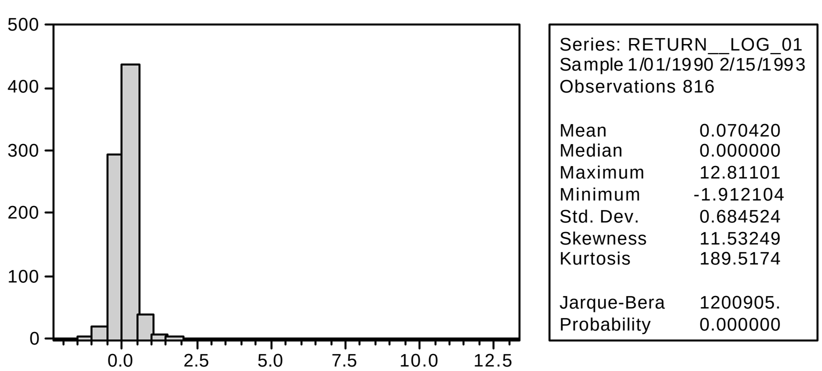
8. Returns for the period from January 2000 to November2002

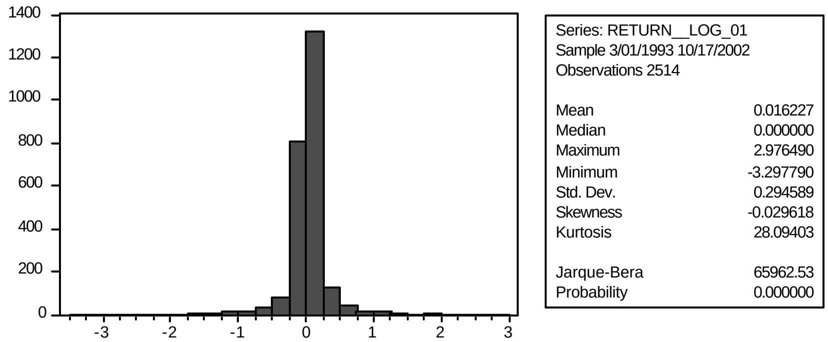


1.

Histograms



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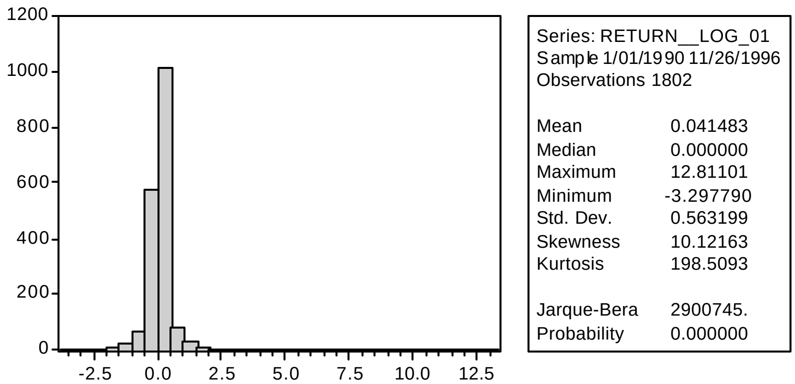
Sample 3/01/1993 10/17/2002
Observations 2514
Mean 0.016227
Median 0.000000
Maximum 2.976490
Minimum -3.297790
Std. Dev. 0.294589
Skewness -0.029618
Kurtosis 28.09403
Jarque-Bera 65962.53
Probability 0.000000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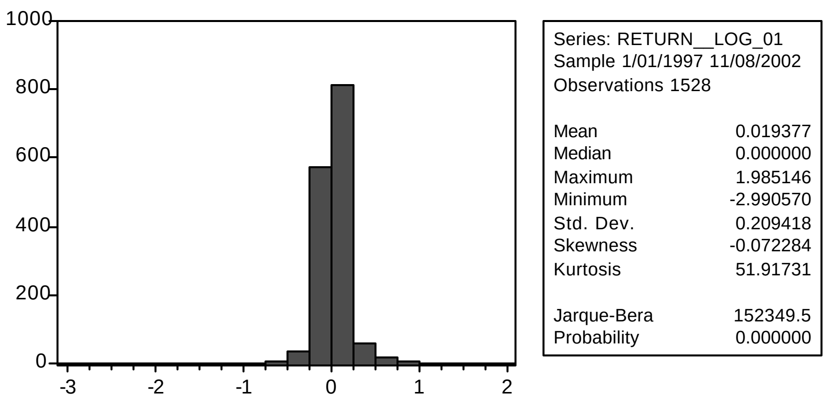
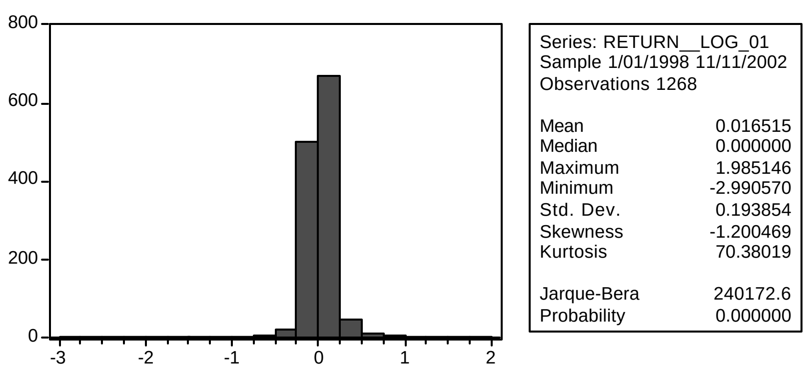
Sample 1/01/1990 11/26/1996
Observations 1802
Mean 0.041483
Median 0.000000
Maximum 12.81101
Minimum -3.297790
Std. Dev. 0.563199
Skewness 10.12163
Kurtosis 198.5093
Jarque-Bera 2900745.
Probability 0.000000


Series: RETURN__LOG_01
Sample 1/01/1997 11/08/2002
Observations 1528
Mean 0.019377
Median 0.000000
Maximum 1.985146
Minimum -2.990570
Std. Dev. 0.209418
Skewness -0.072284
Kurtosis 51.91731
Jarque-Bera 152349.5
Probability 0.000000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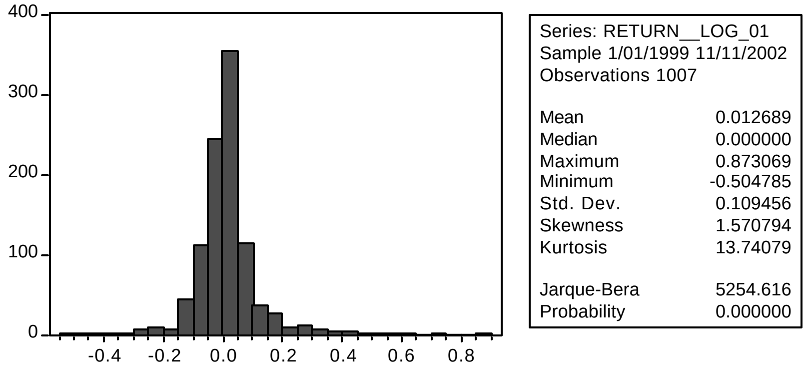
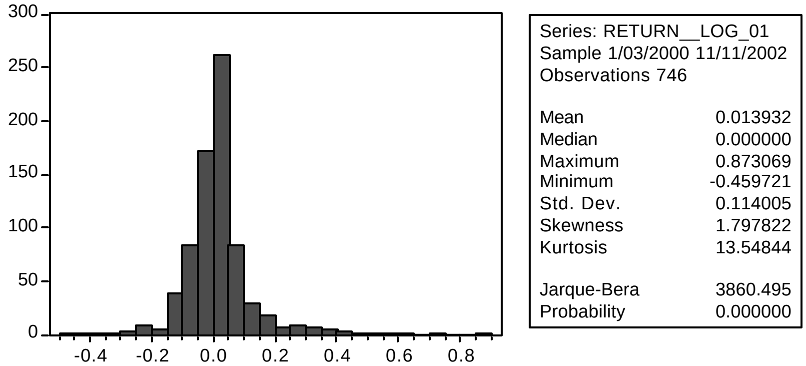
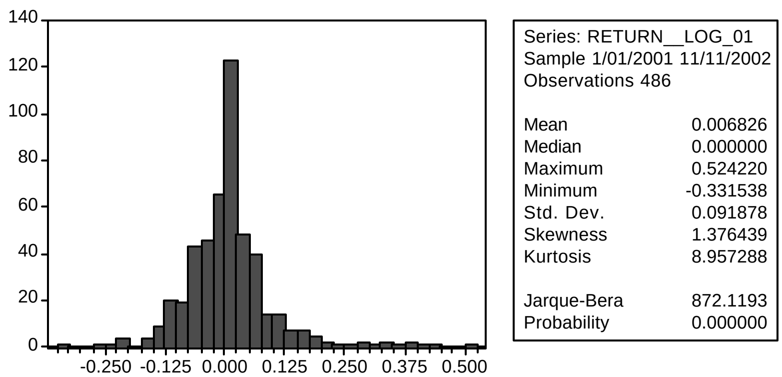
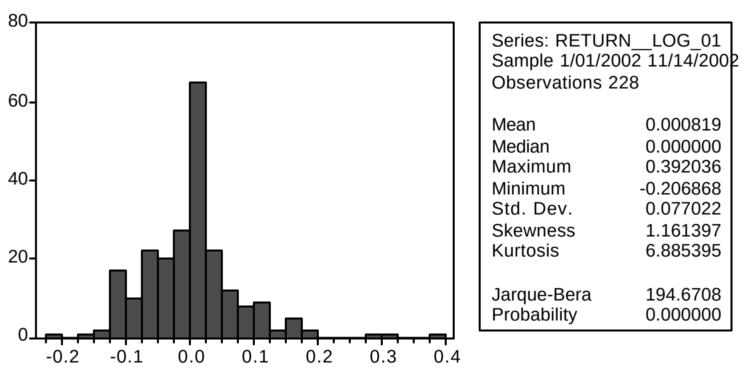
Sample 1/01/1998 11/11/2002
Observations 1268
Mean 0.016515
Median 0.000000
Maximum 1.985146
Minimum -2.990570
Std. Dev. 0.193854
Skewness -1.200469
Kurtosis 70.38019
Jarque-Bera 240172.6
Probability 0.000000



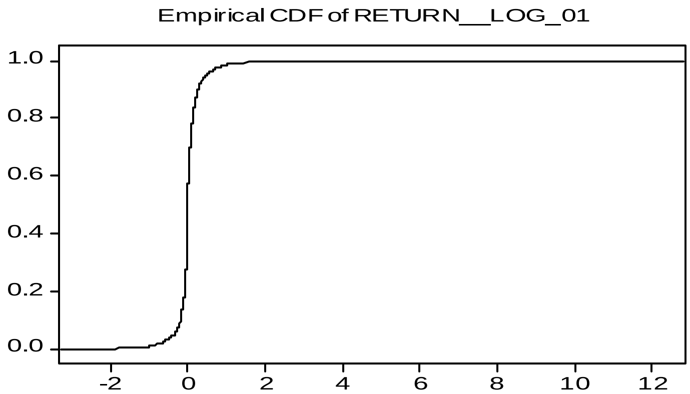




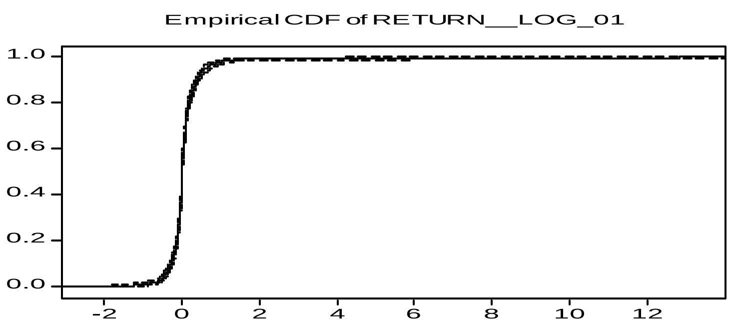


Cumulative Distribution

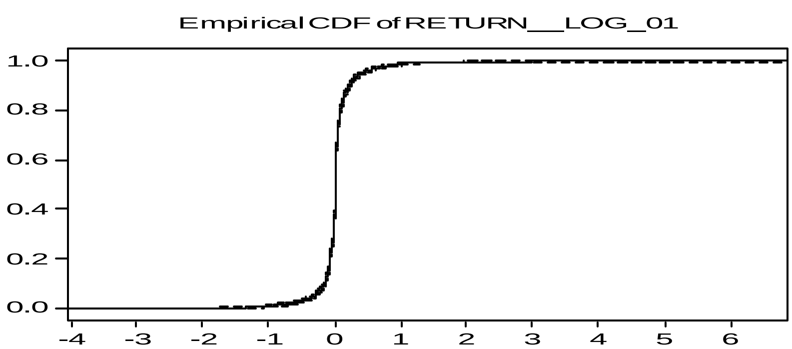
1. CDF for the period from January 199o to November2002



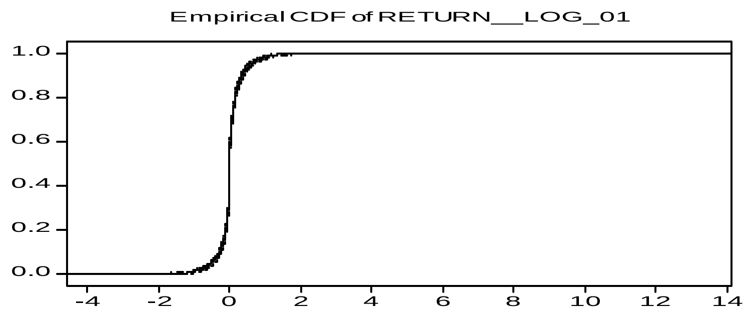
2. CDF for the period from January 1990 to February 1993



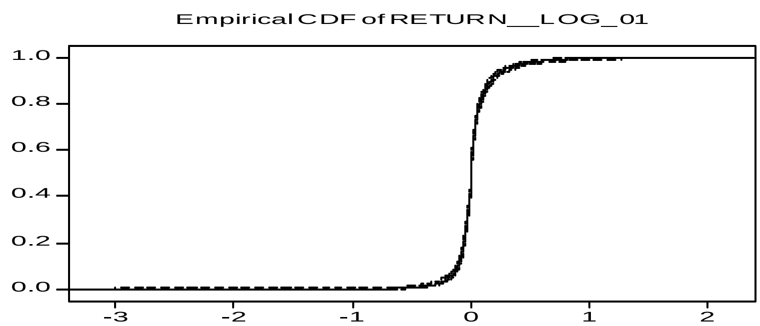
3. CDF for the period from March 1993 to November2002



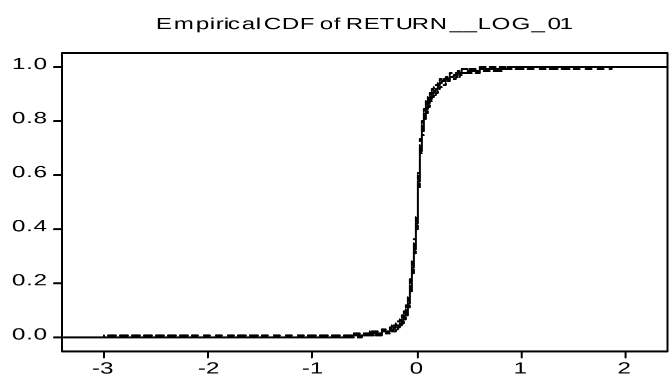
4. CDF for the period from January 19990 to December 1996



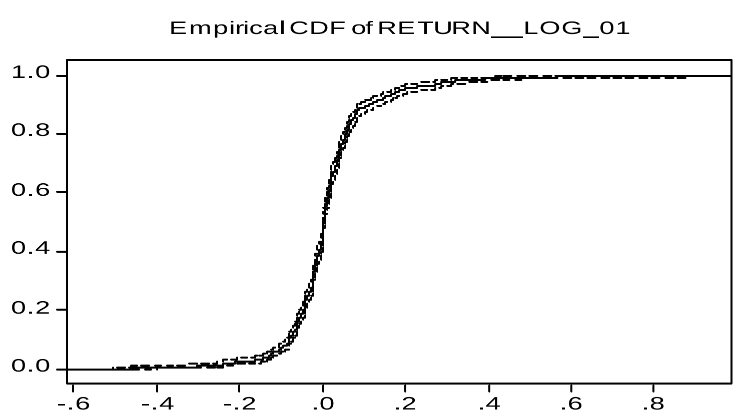
5. CDF for the period from January 1997 to November2002



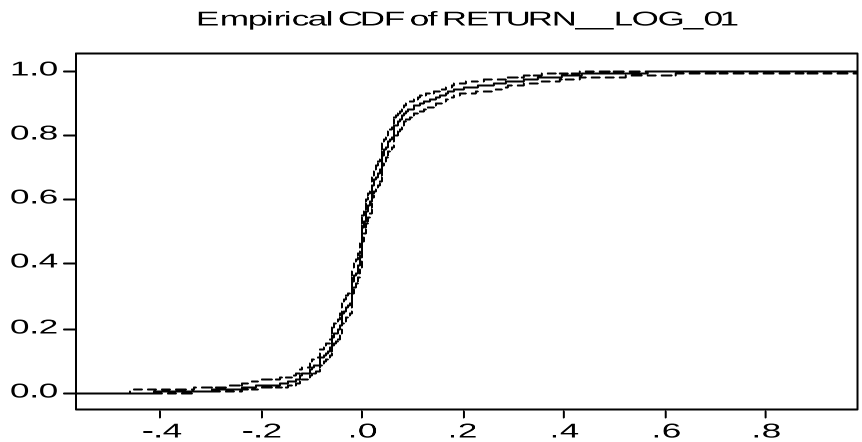
6. CDF for the period from January 1998 to November2002



7. CDF for the period from January 1999 to November2002



8. Returns for the period from January 2000 to November2002



Stationarity Condition Testing:

To use the data for analysis, the time series should be subjected to stationarity condition. To claim that information on the past behavior of an asset's price or returns may be of some value in predicting its future, the implicit assumption is that that there is some *regularity* in the way the random nature of the time series is generated. This also implies that any models that claim to explain this behavior must also possess this fundamental regularity. One way to narrow down what "regularity" means for a random variable over time is the concept of *stationarity*. A time series, X_t, is said to be *weakly stationary* (or *wide-sense stationary* or *covariance stationary*) if it fulfills three properties:

1. Mean is constant over time: $E\left[X_t\right] = m$ for all t.
2. Variance is constant over time: $Var\left[X_t\right] = E\left[\left(X_t - m\right)^2\right] = s_X^2$ for all t.
3. Covariance between any two values of the series depends only on their distance apart in time (k) <u>not</u> on their absolute location in time (t).

$$Cov\left[X_t, X_{t-k}\right] = E\left[\left(X_t - m\right)\left(X_{t-k} - m\right)\right] = g(k)$$

It is possible to relax these requirements further and still do some analysis but it becomes harder. If property 1. does not hold for example, then given n different observations on X, one would have to estimate n different means - one for each period. This means that there are immediately as many unknown parameters as data points, and we have not even worried about variances and covariances yet. By assuming weak stationarity it becomes far simpler to estimate the single mean and variance, and the covariances of interest. Levels of economic and financial time series are generally non-stationary because they exhibit trends over time. Standard procedure is to transform the data, in an intelligent way, so that the result is stationary. This normally involves graphing the levels of the variables of interest against time. If the data appear to lie on a straight line, then first differences of the data (X_t-X_{t-1}) will are generally stationary. If the data lie on an exponential curve, then taking logs of the data and first differencing the logs {i.e. use $z_t=x_t-x_{t-1}$, where $x_t=\ln(X_t)$} generally results in a stationary series.

Time series whose levels or log-levels are stationary are said to be integrated of order 0, termed I(0). Time series whose first-differences are stationary are said to be integrated of order 1, termed I(1). Time series whose k^{th}-differences are stationary are said to be integrated of order k, termed I(k). Most financial time series are either I(0) or I(1). Returns are generally I(0) and asset prices, which under market efficiency follow a Random Walk, are I(1). The D-F equation only tests for first order autocorrelation. If the order is higher, the test is invalid and the D-F equation suffers from residual correlation. It is important to know the order of integration of non-stationary variables, so they may be differenced before being included in a regression equation. The ADF test does this, but it should be noted that it tends to have low power (i.e. it fails to reject H_0 of non-stationarity even when false) against the alternative of a stationary series with ρ near to 1. Phillips – Perron Test also can provide satisfactory results for testing the stationarity conditions.

To test the data series for stationarity condition, the paper has used both ADF and Phillips-Perron Tests. The tests have been performed for Exchange rate close values, their log values and returns.

Augmented Dickey-Fuller Test:

ADF equation: z(t)-z(t-1) = a.z(t-1) + b(1).(z(t-1)-z(t-2)) + ... + b(p).(z(t-p)-z(t-p-1)) + u(t), t = p+2,...,n, where u(t) is white noise. Null hypothesis H(0): z(t) is a unit root process: a = 0. Alternative hypothesis (H1): z(t) is a zero-mean stationary process: a < 0.
The test statistic is the t-value of a. The default lag width is p = [cn^r], where: c = 5 and r = 0.25 (p = 37).

Test Outcomes:

(i) **Close Exchange Rate Values:**

ADF model for z(t)-z(t-1):

	OLS estimate	t-value	Asymptotic critical regions:
z(t-1)	0.0002	2.8269	< -1.93 (5%)
			< -1.60 (10%)
		p-value = 1.00000	

Test result: H0 is not rejected at the 10% significance level

(ii) **Log Values of Exchange Rate**

ADF model for z(t)-z(t-1):

	OLS estimate	t-value	Asymptotic critical regions:
z(t-1)	0.0001	2.8051	< -1.93 (5%)
			< -1.60 (10%)
		p-value = 1.00000	

Test result: H0 is not rejected at the 10% significance level

(iii) **Returns Series (Log Returns)**

ADF model for z(t)-z(t-1):

	OLS estimate	t-value	Asymptotic critical regions:
z(t-1)	-0.6734	-7.8623	< -1.93 (5%)
			< -1.60 (10%)
		p-value = 0.00000	

Test result: H0 is rejected in favor of H1, at the 5% significance level.

ADF test with level, trend and intercept and one lagged difference gives better result and Durbin-Watson stat value is higher satisfying that the series used for analysis is stationary.

ADF Test Statistic	-119.4543	1% Critical Value*	-3.9663
		5% Critical Value	-3.4138
		10% Critical Value	-3.1286

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RETURN__LOG_01,2)
Method: Least Squares
Date: 12/01/02 Time: 06:14
Sample(adjusted): 1/03/1990 10/04/2002
Included observations: 3328 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(RETURN__LOG_01(-1))	-1.622017	0.013579	-119.4543	0.0000
C	0.000314	0.017432	0.018038	0.9856
@TREND(1/01/1990)	-1.28E-07	9.07E-06	-0.014128	0.9887
R-squared	0.811019	Mean dependent var	7.08E-05	

Adjusted R-squared	0.810905	S.D. dependent var	1.155510
S.E. of regression	0.502474	Akaike info criterion	1.462357
Sum squared resid	839.4976	**Schwarz criterion**	**1.467865**
Log likelihood	-2430.362	F-statistic	7134.661
Durbin-Watson stat	**2.361323**	Prob(F-statistic)	0.000000

Phillips-Perron Tests:

Maintained hypothesis: z(t) = a.z(t-1) + b + u(t), where u(t) is a zero-mean stationary process.

Null hypothesis H0: z(t) is a unit root process (a = 1)
Alternative hypothesis H1: z(t) is a stationary process: a < 1.

The test involved is based on n(Alpha - 1), where Alpha is the OLS estimate of the AR parameter 'a'. The test employs a Newey-West type variance estimator of the long-run variance of u(t), with truncation lag m = [c.n^r], where c > 0 and 0 < r < 1/2. The default values of c and r are c = 5, r = .25. However, this test may have low power against the trend stationarity hypothesis. Here m = 37 = [c.n^r], where c=5, r=.25, n=3271

(i) Ex Rate Close:
Alpha = 0.9996
Test statistic: -1.29
p-value = 0.86000
5% Critical region: < -14.51
10% Critical region: < -11.65
Test result: H0 is not rejected at the 10% significance level
(ii) Log Value of Ex Rate:
Alpha = 0.9993
Test statistic: -2.49
p-value = 0.72000
5% Critical region: < -14.51
10% Critical region: < -11.65
Test result: H0 is not rejected at the 10% significance level
(iii) Log Returns:
Alpha = -0.0723
Test statistic: -4365.08
p-value = 0.00000
5% Critical region: < -14.51
10% Critical region: < -11.65
Test result: H0 is rejected in favor of H1, at the 5% significance level.

Both the tests (ADF as well as Phillips-Perron) have revealed that the log return series is stationary while the original time series as well as their log values are non-stationary. Hence the study has used the log returns for the analysis.

Autocorrelation Test Results:

Two common tools exist to identify the autocorrelation structure in a time series: The Autocorrelation Function (ACF) & The Partial Autocorrelation Function (PACF). The ACF indicates the strength of the correlation in a time series between x_t *and* x_{t-k} The PACF describes the correlation between x_t *and* x_{t-k} that isn't explained by lower values of k. For example, is there any left-over correlation at a lag of 2 that isn't explained by the lag 1 relationship? Correlogram with level given in the following table gives us a very satisfactory result.

Included observations: 3329
Q-statistic probabilities adjusted for 1 ARMA term(s)

<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>| |</td><td>| |</td><td>1</td><td>0.013</td><td>0.013</td><td>0.5935</td><td></td></tr>
<tr><td>|* |</td><td>|* |</td><td>2</td><td>0.187</td><td>0.186</td><td>116.56</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>3</td><td>0.015</td><td>0.011</td><td>117.33</td><td>0.000</td></tr>
<tr><td>| |</td><td>*| |</td><td>4</td><td>-0.031</td><td>-0.069</td><td>120.61</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>5</td><td>-0.028</td><td>-0.033</td><td>123.18</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>6</td><td>-0.015</td><td>0.004</td><td>123.96</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>7</td><td>0.031</td><td>0.047</td><td>127.26</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>8</td><td>-0.004</td><td>-0.003</td><td>127.31</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>9</td><td>0.058</td><td>0.042</td><td>138.48</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>10</td><td>-0.001</td><td>-0.005</td><td>138.48</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>11</td><td>0.007</td><td>-0.010</td><td>138.65</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>12</td><td>0.014</td><td>0.016</td><td>139.28</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>13</td><td>-0.024</td><td>-0.019</td><td>141.15</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>14</td><td>0.038</td><td>0.036</td><td>145.86</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>15</td><td>-0.014</td><td>-0.007</td><td>146.54</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>16</td><td>0.008</td><td>-0.008</td><td>146.74</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>17</td><td>0.008</td><td>0.010</td><td>146.93</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>18</td><td>0.019</td><td>0.020</td><td>148.11</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>19</td><td>-0.009</td><td>-0.012</td><td>148.36</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>20</td><td>-0.009</td><td>-0.016</td><td>148.64</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>21</td><td>0.010</td><td>0.011</td><td>148.98</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>22</td><td>0.016</td><td>0.028</td><td>149.88</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>23</td><td>0.026</td><td>0.019</td><td>152.13</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>24</td><td>0.011</td><td>0.000</td><td>152.53</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>25</td><td>-0.023</td><td>-0.035</td><td>154.30</td><td>0.000</td></tr>
<tr><td>| |</td><td>| |</td><td>26</td><td>0.019</td><td>0.018</td><td>155.55</td><td>0.000</td></tr>
</table>

|	|	|	|	27	-0.013	0.003	156.08	0.000
|	|	|	|	28	0.019	0.015	157.30	0.000
|	|	|	|	29	0.009	0.009	157.56	0.000
|	|	|	|	30	0.035	0.027	161.60	0.000
|	|	|	|	31	0.024	0.019	163.59	0.000
|	|	|	|	32	0.027	0.014	165.97	0.000
|	|	|	|	33	0.013	0.004	166.51	0.000
|	|	|	|	34	-0.036	-0.039	170.82	0.000
|	|	|	|	35	-0.037	-0.042	175.34	0.000
|	|	|	|	36	-0.008	0.011	175.57	0.000

There are no significant spikes of ACF or PACFs as shown above which indicates that the residuals of the selected ARIMA(0,1,0) model are white noise, so that there are no significant patterns left in the time series.

The equation for the best model is $Y_t = \Phi_1 + Y_{t-1} + \Phi_0 + e_1$ where $e_1 = Y_t - Y_t$(hat)

ARCH Test:

Heteroskedasticity refers to unequal variance in the regression errors. Heteroskedasticity can arise in a variety of ways and a number of tests have been proposed. Typically a test is designed to test the null hypothesis of homoskedasticity (equal error variance) against some specific alternative heteroskedasticity specification. Many economic time series are nonstationary in mean and variance. Other features that some economic time series exhibit are episodes of unusually high variance which may persist for awhile. One way of modeling these features is to model the variance as well as the series. In forecasting an economic time series, we have seen the importance of using conditional forecasts, for example, one period ahead forecasts conditional on all current and past knowledge. In the same way, if the variance is not constant, conditional forecasts of the variance can be important to the forecaster, especially in situations where risk is important. An example is portfolio analysis where forecasts of the mean return for the holding period as well as the variance for the holding period are critical to the decision maker.

Suppose, for example, that the time series is an AR(1):

$$y(t) = a_0 + a_1\, y(t-1) + e(t)$$

where the error has mean zero and,

$$\hat{e}^2(t) = \alpha_0 + \alpha_1 \hat{e}^2(t-1) + \alpha_2 \hat{e}^2(t-2) + \ldots + WN(t).$$

If the parameters α_1, α_2, etc. are zero then the expected estimated variance is constant or homoskedastic:

$$E_{t-1}[\hat{e}^2(t)] = \alpha_0 .$$

Engle Multiplicative ARCH model takes it one step further. Suppose the error process, e(t) has a multiplicative structure :

$$e(t) = WN(t)\ [\alpha_0 + \alpha_1 e^2(t-1)]$$

where the mean of the white noise series is zero and its variance is one, the white noise and lagged error, $e(t-1)$, are independent, and α_0 is greater than zero and α_1 lies between zero and one. The mean of the error process, $e(t)$, conditional or unconditional will be zero. The error process will not be serially correlated, and its unconditional variance will be constant. However, the conditional variance of the error process will be autoregressive of order one, i.e. ARCH(1).

After conducting the regression on the lagged squared residuals coming of the regression done with lagged values of the log returns of the exchange rate data, we find

R^2 = 0.00002158 & T (number of observations) = 3268

To test if we actually have ARCH in the data we need to do White's ARCH test. And to do the same we need to compute R^2*T where T is the number of observations and R^2 is the R^2 from the regression done on squared errors. The value becomes 0.00002158*3268 = 0.070524 which is less than 3.841, (x^2 – *table)* the 5% critical values with 1 degree of freedom. Hence we can safely say that the series used for the analysis (log returns) has no evidence of ARCH.

(a) Descriptive Statistics: The following table summarises the findings for the dataset that was divided into various time buckets according to their relevance:

Period	Mean	Max	Min	Std dev	Skewness	Kurosis	Jarque-Bera	Counts
1990-02	0.031339	12.81101	-3.29779	0.437995	11.69040	295.8457	11974855	3330
1990-Feb 93	0.070420	12.81101	-1.91210	0.684524	11.53249	189.5174	1200905	816
Mar93-2002	0.016227	2.976490	-3.297790	0.294589	-0.029618	28.09403	65962.53	2514
1990-1996	0.041483	12.81101	-3.29779	0.563199	10.12183	198.5093	2900745	1802
1996-2002	0.017684	2.976480	-2.990570	0.255597	0.745488	39.06626	97127.57	1789
1997-2002	0.019377	1.985146	-2.990570	0.209418	-0.072284	51.91731	152349.5	1529
1998-2002	0.016515	1.985146	-2.990570	0.193854	-1.200469	70.38019	240172.6	1268
1999-2002	0.012689	0.873069	-0.504785	0.109456	1.570794	13.74079	5154.616	1007
2000-2002	0.013932	0.873069	-0.459721	0.114005	1.797822	13.54844	3860.495	746
2001-2002	0.006826	0.524220	-0.331538	0.091878	1.376439	8.957228	972.1193	486
2002-2002	0.000819	0.392036	-0.206868	0.077022	1.161397	6.885395	194.6708	228

Now let us concentrate to see if the trends indicate the persistence of long memory in INR/USD exchange rate returns using two important methodologies.

Methodologies used for Persistence Tests

If asset prices display long memory, or long-term dependence, they exhibit significant autocorrelation between observations widely separated in time. This implies that what has happened not only in recent past but long time back has a bearing on the today's market prices and hence existence of an autocorrelation between these observations. Today's risk containment policies followed in Indian financial markets, specifically by banks whio have a very large stake in the foreign exchange market, are built on the basis of historical price behaviour (VaR on the basis of historical simulation). Since the series realizations are not independent over time, realizations from the remote past can help us predict future movements in asset prices. Persistence in returns has a special claim on the attention of investors because any predictable trend in returns should be readily exploitable by an appropriate strategy.

Peters (1989) used Hurst Rescaled Range (R/S) analysis to measure non-periodic cycles in financial data. He concluded that capital market prices do not reflect information immediately, as the efficient market hypothesis assumes, but rather follow a biased random walk that reflects persistence. Using the rescaled range (R/S) method, Greene and Fielitz (1977) also reported evidence of persistence in daily U.S. stock returns series. Barkoulas and Baum used Spectral Regression Test to test the long memory of US stocks and found only few stock do have long memory. However, according to some experts, the classical R/S test is biased toward finding long-term memory too frequently. Financial asset returns may follow biased time paths that standard statistical tests cannot distinguish from random behavior. Rescaled range analysis can be used to detect long-term, non-periodic cycles in stock market returns. If this technique is not applied correctly, however, then it can be influenced by short-term biases, leading to the erroneous conclusion that the market has long-term memory.

The primary focus of these studies has been the stochastic long memory behaviour of stock returns in major capital markets. In contrast, the long memory behaviour in smaller markets has received little attention. Contrary to findings for major capital markets, Barkoulas, Baum, and Travlos in a Working Paper found significant and robust evidence of positive long-term persistence in the Greek stock market. Wright (1999) used AFRIMA model to test long memory in emerging market including India and came up with the conclusion that emerging markets appear to have considerable serial correlation which stands contrast to the results for the developed markets like US, where there is little evidence for any serial correlation in stock returns.

Today, we see an overwhelming response to emerging markets from investors across the world. These markets have provided diversification opportunity to international investors. It must be noted that such markets are very likely to exhibit characteristics different from those observed in developed markets as the market micro-structure is different in emerging markets vis-à-vis developed ones. Biases due to market thinness and non-synchronous trading, regulatory intervention should be expected to be more severe in the case of the emerging markets. In case of Indian foreign exchange market, we have seen on many occasions spikes in exchange rate behaviour not because of the true market conditions but due to some sort of administrative actions taken by the central bank but over the years their intervention has come down. Another important factor that need to be considered is that these emerging markets have been going through many regulatory changes to improve the efficiency level and can not be fully compared with the developed and established markets.

In this study, we look for evidence of long memory in Rupee-Dollar exchange rate. We have used data about returns from the Exchange Rate data released by RBI. to check for persistence of long memory in daily returns data. The exchange rate data is the mid-rate disseminated by RBI on daily basis as a reference rate. Two methods have been used for the test: (a) the Variance Ratio Test and (b) the Hurst Exponent (R/S Analysis) to test the data.

Variance Ratio Test

The first test, which I consider, is the Variance Ratio Test popularized by Cochrane (1988), and used by MacDonald and Power (1992) etc.

$$Z(k) = (1/k) * \{ \mathrm{Var}(X_{t-k}) / \mathrm{Var}(X_t) \}$$

Where X_t denotes a one period return, obtained from the first difference of the natural logarithmic of the exchange rate, and X_{t-k} denotes the *k*-period return calculated using the *k*th difference of the exchange rate. The possibilities are as follows:

1. If the price series follows a random walk, this ratio should equal unity.
2. If the series is stationary, the ratio will tend to zero.
3. If price exhibit mean reversion, Z (*k*) should lie between zero and one.
4. Values of Z (*k*) above one indicate that a current increase in the value of the price will be reinforced in the future by further positive increases.

Performing the above analysis on the about 12 years exchange rate data, following results were obtained:

Lag	**1 Day**	**15 days**	**30 days**	**90 days**	**180 days**	**270 days**	**360days**	**720 days**	**1800days**
Variance	0.19184	2.17967	5.20048	17.2949	38.9201	61.28011	62.5922	173.8514	164.6052
Variance Ratio		0.5961	0.9036	1.0017	1.1271	1.1831	0.9063	1.2587	0.4767

The above tests show some interesting results. The above results show that in the 3, 6 months, 9 months and 2 years lag, the variance ratio has been greater than 1 that indicates the persistence or a trend-reinforcing tendency in exchange rate returns. In the lag periods of 15 days, 30 days, 1 year and 5 years the same is between 0 to 1 indicating mean reversion tendency. However, the values for 1 month and 1 year have been very close to 1 indicating there may be noise in the series.

R/S Analysis

Standard autocorrelation tests detect long-term dependency in stock market prices if dependent behaviour is periodic and if the periodicity is consistent over time. Fundamental historical changes however alter the period of cycles. Mandelbrot (1972) proposes a statistic to measure the degree of long-term dependency, in particular, “non periodic cycles”. The rescaled range, or R/S statistic, is formed by measuring the range between the maximum and minimum distances that the cumulative sum of a stochastic random variable has strayed from its mean and then dividing this by its standard deviation. An unusually small R/S measure would be consistent with mean reversion, for instance, while an unusually large one would be consistent with return persistence. To construct this statistic, consider a sample of returns $X_1, X_2, \ldots, X_n$ and let X denote the sample mean.

$$Q_n = 1/(\sigma_n . \sqrt{n}) [\max \Sigma (X_j - X_n) - \min \Sigma (X_j - X_n)]$$

In his original work, Mandelbrot suggested using the sample standard deviation estimator for the scaling factor, σ_n

We have used this as my second technique to judge persistence in stock market returns. Peters (1994) has discussed this method in a simpler and neater way. Let us take a series of data $X_1, X_2, \ldots, X_n$ and let X denote the sample mean. Let σ_n again be the standard deviation. The rescaled range was calculated by first rescaling or "normalizing" the data by subtracting the sample mean:

$$Z_r = X_r - X \qquad r = 1,2\ldots\ldots n$$

The resulting series, Z, now has a mean of zero. The next step creates a cumulative time series Y:

$$Y_1 = Z_1 + Z_r \qquad r = 2,3\ldots\ldots n$$

Note that by definition the last value of Y (Y_n) will always be zero because Z has a mean of zero.

The adjusted range, R_n is the maximum minus the minimum value of the Y_r:

$$R_n = \max (Y_1 ,\ldots\ldots\ldots, Y_n) - \min (Y_1 ,\ldots\ldots\ldots, Y_n)$$

The subscript n for R_n now signifies that this is the adjusted range for $X_1, X_2, \ldots, X_n$. Because Y has been adjusted to a mean of zero, the maximum value of Y will always be greater than or equal to zero, and the minimum will always be less than or equal to zero. Hence, the adjusted range R_n will always be non-negative.

However, this equation applies only to time series in Brownian motion: that have mean zero and variance equal to one. To apply to any time series (like stock returns), we need to generalize the equation. Hurst found that the following was a more general form of equation:

$$R/\sigma = c * n^H$$

The R/S (or R/σ) value is referred to as the rescaled range analysis because it has mean zero and is expressed in terms of local standard deviation. In general, the R/S value scales as we increase the time increment, n, by a "power–law "value equal to H, generally called the Hurst exponent.

The procedure used for calculations is listed below (Peters, 1994; pp. 62-63).

1. Begin with a time series of length M. Convert this into a time series of length

N = (M – 1) of logarithmic ratios: $N_i = \log(M_{i+1} / M_i)$, I = 1,2,3, …….., (M-1)

2. Divide this time period into A contiguous sub periods of length n, such that A*n = N. Label each sub period I_a with a = 1,2,3,….A. Each element in I_a is labeled $N_{k,a}$ such that k = 1,2,3,…n. For each I_a of length n, the average value is defined as: $e_n = (1/n) * \Sigma N_{k,a}$ where e_n = average value of the Ni contained in sub period I_a of length n.

3. The time series of accumulated departures ($X_{k,a}$) from the mean value for each sub period I_a is defined as $X_{k,a} = (N_{i,a} – e_a)$ where k = 1,2,3, ….n

4. The range is defined as the maximum minus the minimum value of $X_{k,a}$ within each sub period I_a: $R_{Ia} = \max(X_{k,a}) – \mathrm{Min}(X_{k.a})$ where 1<= k<= n

5. The sample standard deviation calculated for each sub period I_a: $S_{Ia} = (1/n) * \Sigma(N_{i,a} – e_a^2)^{0.50}$

6. Each range R_{Ia} is now normalized by dividing the S_{Ia} corresponding to it. Therefore, the rescaled range for each I_a sub period is equal to R_{Ia} / S_{Ia}. From step 2, we had A contguous sub periods of length n. Therefore, the average R/S value for length n is defined as $(R/S)n = (1/A) * \Sigma (R_{Ia} / S_{Ia})$

7. The length n is increased to the next higher value, and (M-1) /n is an integer value. We use values of n that include the beginning and ending points of the time series, and steps 1 through 6 are repeated until n = (M-1)/2.

Hurst's Empirical Law

Hurst (1951) also gave a formula for estimating the value of H from a single R/S value (as quoted in Peters, 1996):

$$H = \log (R/S) / \log (n/2)$$
where n = number of observations

This equation assumes that the constant c of the above equation is equal to 0.5. Feder (1988) shows that the empirical law tends to overstate H when it is greater than 0.70 and understate it when it is less than or equal to 0.40. However for short data sets, where regression is not possible, the empirical law can be used as a reasonable estimate.

The method discussed above would become clearer by looking at the calculations done for exchange rate data. We can use daily exchange rate closing data for the last decade (i.e. 1990 to 2002) and calculate daily logarithmic returns. The same data has been used to estimate H for 4 periods of time (with N=2, 15, 30, 90, 180,270, 360, 720 & 1800) since plotting a line between log (R/S) vs log (N) is very difficult. There are 3 distinct classifications for the Hurst exponent (H):

1. H = 0.5

H equal to 0.5 denotes a random series.

2. 0<=H < 0.5

This type of system is anti persistent or mean reverting. That means if the system has been up in the previous period, it is likely to be down in the next period. The strength of anti-persistent behaviour will depend on how close H is to 0.

3. 0.5 < H <1.0

Here we have a persistent or trend reinforcing series. That means, if the series has been up (down) in the last period, hence the chances are that it will continue to be positive(negative) in the next period. Trends are apparent. The strength of the trend-reinforcing behaviour, or persistence, increases as H approaches 1. The closer H is to 0.5, the noisier it will be and the trend would be less defined. Persistent series are fractional Brownian motion, or biased random walks. The strength of the bias depends on how far H is above 0.50.

The results for the Exchange rate data are listed below:

Period, N	**2 Day**	**15 days**	**30 days**	**90 days**	**180 days**	**270 days**	**360days**	**720 days**	**1800days**
Hurst Exponent	-	**0.523169**	**0.525509**	**0.537446**	**0.533630**	**0.569032**	**0.585479**	**0.583056**	**0.620881**

This shows that there is a definite possibility for persistence in the exchange rate returns data but the values are very close to 0.50 leading us to believe that there is enough noise in the series and the trend is not perfectly established.

Conclusion

The normality tests on the daily exchange rate returns for the last one-decade or so indicate the need to explore the application of non-linear modeling techniques while understanding exchange rate behaviour. But we come to see that the results from the persistence tests are split. The variance test clearly implies that there exists only short-term memory in the market returns as given by study above and the pattern is also not clearly established. However, the R/S analysis does give indications of long-term memory but with noise. In either case, analysis shows that the movement of exchange rate does not follow a random movement. However, a more rigid analysis needs to be performed, maybe by using Lo's modified R/S Analysis. Also, for a foolproof analysis, the data used should be for a period longer than just one decade.

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