Reproduction of some key results in Leland et al.

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This short monograph describes a reproduction of some of the key results in the famous Leland et al. Bellcore study of Ethernet traffic (see [1]). The results herein were all generated using the traffic analysis tools available at [3]. The raw data for the Bellcore study (see [2]) is stored in text file format as *<time-stamp, packet size>* pairs. Figure 1 shows the first dozen entries from file `pAugTL.z`.

```
0.001340  1090
0.001508   174
0.004176   162
0.008140   174
0.011036   162
0.015072   174
0.017892   162
0.020604   150
0.022032   174
0.024300    90
0.024752   162
0.027356   150
```

Figure 1 - The raw Bellcore data (first dozen entries in `pAugTL.z`)

The time stamps of the Bellcore raw data are cumulative. This raw data was stripped of the packet size information and then converted to packet counts per 0.010 seconds. The tool used to convert cumulative time stamps to packet counts was `ttoc1.c`. The converted packet count file was named `bell.dat` and contained 314283 count values. Figure 2 shows the batch file used to compute autocorrelation values for `bell.dat` and mean, variance, and R/S values for `bell.dat` and aggregations of 10x, 100x, and 1000x. The programs `block1.c`, `mean1.c`, `var1.c`, `autoc1.c`, and `hurst1.c` are all available at [3]. The program `hurst1.c` computes a single R/S value for a series *X*.

```
block1 < bell.dat       > b10.dat
block1 < b10.dat        > b100.dat
block1 < b100.dat       > b1000.dat
mean1  < bell.dat       > results
mean1  < b10.dat       >> results
mean1  < b100.dat      >> results
mean1  < b1000.dat     >> results
var1   < bell.dat      >> results
var1   < b10.dat       >> results
var1   < b100.dat      >> results
var1   < b1000.dat     >> results
autoc1 < bell.dat      >> results
hurst1 < bell.dat      >> results
hurst1 < b10.dat       >> results
hurst1 < b100.dat      >> results
hurst1 < b1000.dat     >> results
```

Figure 2 - Batch file for analysis of `bell.dat`

The Hurst parameter (*H*) is estimated from the slope of the line of a plot of Log(*M*) versus Log(*R/S*). In this case, *M* = 1000 for the *R/S* value of `bell.dat`, *M* = 100 for `b10.dat`, *M* = 10 for `b100.dat`, and *M* = 1 for `b1000.dat`. The analysis of Figure 2 on `bell.dat` was repeated on a file `poisson.dat` where `poisson.dat` contained 314283 Poisson distributed values of mean 3.184662 (3.181846 is the mean value of `bell.dat`). The values in `poisson.dat` were generated using the random number generator in Microsoft Excel with a Lambda value of 3.181846. The execution time for the batch file of Figure 2 was about 170 minutes on a 100-MHz Pentium PC with 16-Mbytes of RAM. Of the total 170 minutes, 164 minutes was for execution of `hurst1 < bell.dat >> results`.

Table 1 show the mean and variance for the Bellcore data and the Poisson distributed values. Figure 3 shows the autocorrelation for `bell.dat` and `poisson.dat` for the first 1000 lags. Finally, Figure 4 shows the plotted Log(*M*) versus Log(*R/S*) for the Bellcore data and the Poisson distributed values. Shown on the graphs are the equation of the fitted line and the R^2 value of the least squares fit. We note the estimated *H* values from the slope of the fitted line, *H* = 0.52 for Poisson and *H* = 0.84 from the Bellcore data. In [1] an *H* value of 0.85 is reported for the Bellcore data and *H* = 0.50 is expected for Poisson values. The raw data (i.e., the `results` file from the Figure 2 batch file execution), slightly abridged, for these Figures is included in Appendix A and B.

	Bellcore data		Poisson values	
Log(M)	Mean	Variance	Mean	Variance
3	3.181846	6.80294	3.184662	3.177
2	3.181857	3.235453	3.18467	0.319095
1	3.181706	1.3108	3.184704	0.031745
0	3.181787	0.638428	3.184564	0.003338

Table 1 - Mean and variance for Bellcore data and Poisson distributed values

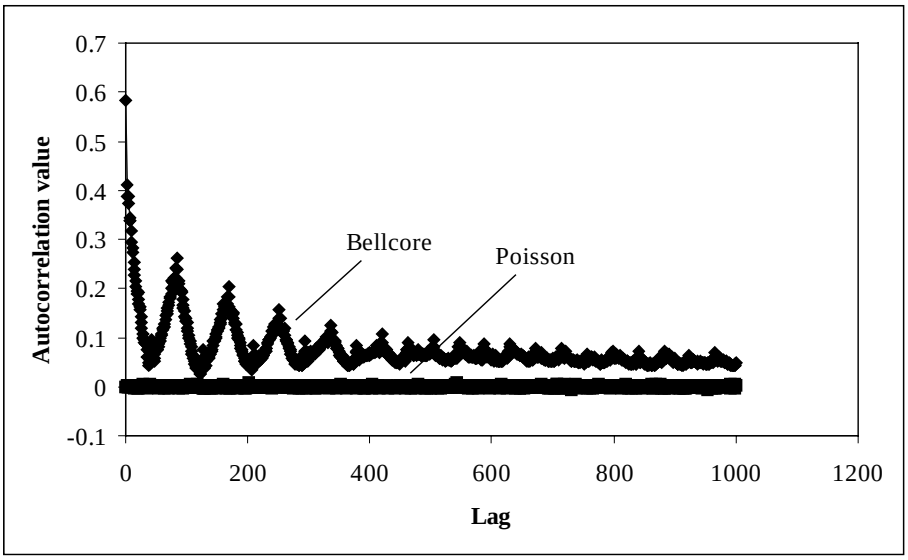

Figure 3 - Autocorrelation for the Bellcore data and Poisson distributed values

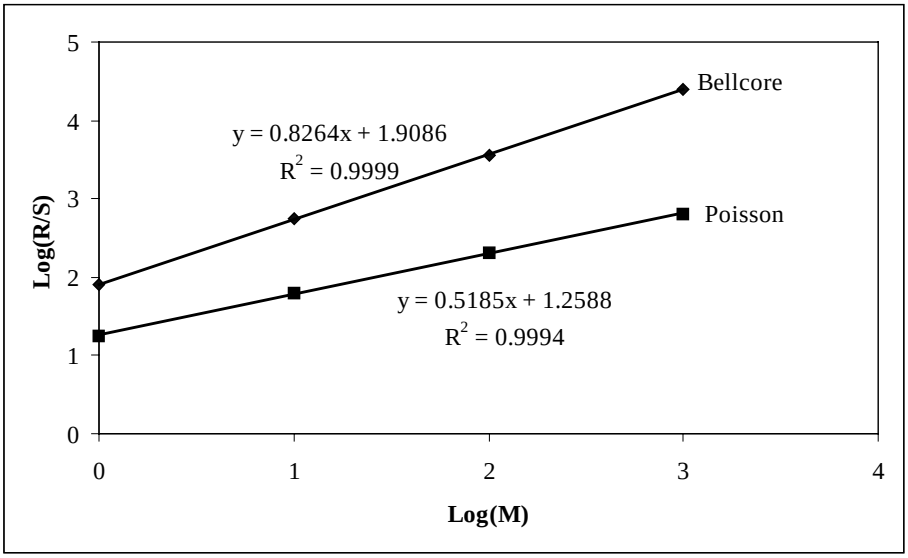

Figure 4 - Hurst parameter estimate for the Bellcore data and Poisson distributed values

References

[1] W. Leland, M. Taqqu, W. Willinger, and D. Wilson, "On the Self-Similar Nature of Ethernet Traffic (Extended Version)," *IEEE/ACM Transactions on Networking*, Vol. 2, No. 1, pp. 1 - 15, February 1994.

[2] BC - 4 million-packet traces of LAN and WAN traffic seen on an Ethernet. URL: *http://ita.ee.lbl.gov/html/contrib/BC.html*.

[3] Tools page for Kenneth J. Christensen, 1998. URL: *http://www.csee.usf.edu/~christen/toolpage.html*.

Appendix A - Raw data for Bellcore data analysis

```
------------------------------------------------- mean1.c -----
  Mean for 314283 values = 3.181846
-------------------------------------------------------------
------------------------------------------------- mean1.c -----
  Mean for 31428 values = 3.181857
-------------------------------------------------------------
------------------------------------------------- mean1.c -----
  Mean for 3142 values = 3.181706
-------------------------------------------------------------
------------------------------------------------- mean1.c -----
  Mean for 314 values = 3.181787
-------------------------------------------------------------
------------------------------------------------- var1.c -----
  Variance for 314283 values = 6.802940
-------------------------------------------------------------
------------------------------------------------- var1.c -----
  Variance for 31428 values = 3.235453
-------------------------------------------------------------
------------------------------------------------- var1.c -----
  Variance for 3142 values = 1.310800
-------------------------------------------------------------
------------------------------------------------- var1.c -----
  Variance for 314 values = 0.638428
-------------------------------------------------------------
------------------------------------------------ autoc1.c -----
  Autocorrelation for lag    1 = 0.582134
  Autocorrelation for lag    2 = 0.412211
  Autocorrelation for lag    3 = 0.388460
  Autocorrelation for lag    4 = 0.388352
  Autocorrelation for lag    5 = 0.373063
  Autocorrelation for lag    6 = 0.345029

  <SNIP SNIP>

  Autocorrelation for lag  997 = 0.044254
  Autocorrelation for lag  998 = 0.043458
  Autocorrelation for lag  999 = 0.045363
  Autocorrelation for lag 1000 = 0.050316
-------------------------------------------------------------
------------------------------------------------ hurst1.c -----
  R/S = 24566.823936 for series X of 314283 values
-------------------------------------------------------------
------------------------------------------------ hurst1.c -----
  R/S = 3562.161561 for series X of 31428 values
-------------------------------------------------------------
------------------------------------------------ hurst1.c -----
  R/S = 559.144809 for series X of 3142 values
-------------------------------------------------------------
------------------------------------------------ hurst1.c -----
  R/S = 80.093527 for series X of 314 values
-------------------------------------------------------------
```

Appendix B - Raw data for Poisson values analysis

```
------------------------------------------------- mean1.c -----
  Mean for 314283 values = 3.184662
-------------------------------------------------------------
------------------------------------------------- mean1.c -----
  Mean for 31428 values = 3.184670
-------------------------------------------------------------
------------------------------------------------- mean1.c -----
  Mean for 3142 values = 3.184704
-------------------------------------------------------------
------------------------------------------------- mean1.c -----
  Mean for 314 values = 3.184564
-------------------------------------------------------------
------------------------------------------------- var1.c -----
  Variance for 314283 values = 3.177000
-------------------------------------------------------------
------------------------------------------------- var1.c -----
  Variance for 31428 values = 0.319095
-------------------------------------------------------------
------------------------------------------------- var1.c -----
  Variance for 3142 values = 0.031745
-------------------------------------------------------------
------------------------------------------------- var1.c -----
  Variance for 314 values = 0.003338
-------------------------------------------------------------
------------------------------------------------ autoc1.c -----
  Autocorrelation for lag    1 = 0.000109
  Autocorrelation for lag    2 = 0.000225
  Autocorrelation for lag    3 = 0.001268
  Autocorrelation for lag    4 = -0.001103
  Autocorrelation for lag    5 = -0.000604
  Autocorrelation for lag    6 = 0.001608

  <SNIP SNIP>

  Autocorrelation for lag  997 = 0.001854
  Autocorrelation for lag  998 = 0.003876
  Autocorrelation for lag  999 = -0.003065
  Autocorrelation for lag 1000 = 0.001379
-------------------------------------------------------------
------------------------------------------------ hurst1.c -----
  R/S = 636.297316 for series X of 314283 values
-------------------------------------------------------------
------------------------------------------------ hurst1.c -----
  R/S = 200.076903 for series X of 31428 values
-------------------------------------------------------------
------------------------------------------------ hurst1.c -----
  R/S = 62.852088 for series X of 3142 values
-------------------------------------------------------------
------------------------------------------------ hurst1.c -----
  R/S = 17.496715 for series X of 314 values
-------------------------------------------------------------
```