Reproduction of some key results in Leland et al.

Kenneth J. Christensen
University of South Florida
christen@csee.usf.edu

This short monograph describes a reproduction of some of the key results in the famous Leland et al. Bellcore study of Ethernet traffic (see [1]). The results herein were all generated using the traffic analysis tools available at [3]. The raw data for the Bellcore study (see [2]) is stored in text file format as \( \langle \text{time-stamp, packet size} \rangle \) pairs. Figure 1 shows the first dozen entries from file pAugTL.z.

\[
\begin{align*}
0.001340 & \quad 1090 \\
0.001508 & \quad 174 \\
0.004176 & \quad 162 \\
0.008140 & \quad 174 \\
0.011036 & \quad 162 \\
0.015072 & \quad 174 \\
0.017892 & \quad 162 \\
0.020604 & \quad 150 \\
0.022032 & \quad 174 \\
0.024300 & \quad 90 \\
0.024752 & \quad 162 \\
0.027356 & \quad 150 \\
\end{align*}
\]

**Figure 1** - The raw Bellcore data (first dozen entries in pAugTL.z)

The time stamps of the Bellcore raw data are cumulative. This raw data was stripped of the packet size information and then converted to packet counts per 0.010 seconds. The tool used to convert cumulative time stamps to packet counts was ttocl.c. The converted packet count file was named bell.dat and contained 314283 count values. Figure 2 shows the batch file used to compute autocorrelation values for bell.dat and mean, variance, and R/S values for bell.dat and aggregations of 10x, 100x, and 1000x. The programs block1.c, mean1.c, var1.c, autoc1.c, and hurst1.c are all available at [3]. The program hurst1.c computes a single R/S value for a series \( X \).

\[
\begin{align*}
\text{block1} & \quad \text{< bell.dat} & \quad \text{> b10.dat} \\
\text{block1} & \quad \text{< b10.dat} & \quad \text{> b100.dat} \\
\text{block1} & \quad \text{< b100.dat} & \quad \text{> b1000.dat} \\
\text{mean1} & \quad \text{< bell.dat} & \quad \text{> results} \\
\text{mean1} & \quad \text{< b10.dat} & \quad \text{> results} \\
\text{mean1} & \quad \text{< b100.dat} & \quad \text{> results} \\
\text{mean1} & \quad \text{< b1000.dat} & \quad \text{> results} \\
\text{var1} & \quad \text{< bell.dat} & \quad \text{> results} \\
\text{var1} & \quad \text{< b10.dat} & \quad \text{> results} \\
\text{var1} & \quad \text{< b100.dat} & \quad \text{> results} \\
\text{var1} & \quad \text{< b1000.dat} & \quad \text{> results} \\
\text{autoc1} & \quad \text{< bell.dat} & \quad \text{> results} \\
\text{hurst1} & \quad \text{< bell.dat} & \quad \text{> results} \\
\text{hurst1} & \quad \text{< b10.dat} & \quad \text{> results} \\
\text{hurst1} & \quad \text{< b100.dat} & \quad \text{> results} \\
\text{hurst1} & \quad \text{< b1000.dat} & \quad \text{> results} \\
\end{align*}
\]

**Figure 2** - Batch file for analysis of bell.dat
The Hurst parameter \((H)\) is estimated from the slope of the line of a plot of \(\log(M)\) versus \(\log(R/S)\). In this case, \(M = 1000\) for the \(R/S\) value of bell.dat, \(M = 100\) for b10.dat, \(M = 10\) for b100.dat, and \(M = 1\) for b1000.dat. The analysis of Figure 2 on bell.dat was repeated on a file poisson.dat where poisson.dat contained 314283 Poisson distributed values of mean 3.184662 (3.181846 is the mean value of bell.dat). The values in poisson.dat were generated using the random number generator in Microsoft Excel with a Lambda value of 3.181846. The execution time for the batch file of Figure 2 was about 170 minutes on a 100-MHz Pentium PC with 16-Mbytes of RAM. Of the total 170 minutes, 164 minutes was for execution of hurst1 < bell.dat >> results.

Table 1 show the mean and variance for the Bellcore data and the Poisson distributed values. Figure 3 shows the autocorrelation for bell.dat and poisson.dat for the first 1000 lags. Finally, Figure 4 shows the plotted \(\log(M)\) versus \(\log(R/S)\) for the Bellcore data and the Poisson distributed values. Shown on the graphs are the equation of the fitted line and the \(R^2\) value of the least squares fit. We note the estimated \(H\) values from the slope of the fitted line, \(H = 0.52\) for Poisson and \(H = 0.84\) from the Bellcore data. In [1] an \(H\) value of 0.85 is reported for the Bellcore data and \(H = 0.50\) is expected for Poisson values. The raw data (i.e., the results file from the Figure 2 batch file execution), slightly abridged, for these Figures is included in Appendix A and B.

<table>
<thead>
<tr>
<th>Log(M)</th>
<th>Bellcore data</th>
<th>Poisson values</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.181846</td>
<td>3.184662</td>
</tr>
<tr>
<td>2</td>
<td>3.181857</td>
<td>3.18467</td>
</tr>
<tr>
<td>1</td>
<td>3.181706</td>
<td>3.184704</td>
</tr>
<tr>
<td>0</td>
<td>3.181787</td>
<td>3.184564</td>
</tr>
</tbody>
</table>

Table 1 - Mean and variance for Bellcore data and Poisson distributed values

![Figure 3 - Autocorrelation for the Bellcore data and Poisson distributed values](image-url)
Figure 4 - Hurst parameter estimate for the Bellcore data and Poisson distributed values

References


Appendix A - Raw data for Bellcore data analysis

Mean for 314283 values = 3.181846
Mean for 31428 values = 3.181857
Mean for 3142 values = 3.181706
Mean for 314 values = 3.181787

Variance for 314283 values = 6.802940
Variance for 31428 values = 3.235453
Variance for 3142 values = 1.310800
Variance for 314 values = 0.638428

Autocorrelation for lag 1 = 0.582134
Autocorrelation for lag 2 = 0.412211
Autocorrelation for lag 3 = 0.388460
Autocorrelation for lag 4 = 0.388352
Autocorrelation for lag 5 = 0.373063
Autocorrelation for lag 6 = 0.345029
Autocorrelation for lag 997 = 0.044254
Autocorrelation for lag 998 = 0.043458
Autocorrelation for lag 999 = 0.045363
Autocorrelation for lag 1000 = 0.050316

R/S = 24566.823936 for series X of 314283 values
R/S = 3562.161561 for series X of 31428 values
R/S = 559.144809 for series X of 3142 values
R/S = 80.093527 for series X of 314 values
Appendix B - Raw data for Poisson values analysis

----------------------------------------------- mean1.c -----
Mean for 314283 values = 3.184662
-----------------------------------------------
Mean for 31428 values = 3.184670
-----------------------------------------------
Mean for 3142 values = 3.184704
-----------------------------------------------
Mean for 314 values = 3.184564
-----------------------------------------------
----------------------------------------------- var1.c -----
Variance for 314283 values = 3.177000
-----------------------------------------------
Variance for 31428 values = 0.319095
-----------------------------------------------
Variance for 3142 values = 0.031745
-----------------------------------------------
Variance for 314 values = 0.003338
-----------------------------------------------
----------------------------------------------- autoc1.c -----
Autocorrelation for lag  1 = 0.000109
Autocorrelation for lag  2 = 0.000225
Autocorrelation for lag  3 = 0.001268
Autocorrelation for lag  4 = -0.001103
Autocorrelation for lag  5 = -0.000604
Autocorrelation for lag  6 = 0.001608
<SNIP SNIP>
Autocorrelation for lag 997 = 0.001854
Autocorrelation for lag 998 = 0.003876
Autocorrelation for lag 999 = -0.003065
Autocorrelation for lag 1000 = 0.001379
-----------------------------------------------
----------------------------------------------- hurst1.c -----
R/S = 636.297316 for series X of 314283 values
-----------------------------------------------
R/S = 200.076903 for series X of 31428 values
-----------------------------------------------
R/S = 62.852088 for series X of 3142 values
-----------------------------------------------
R/S = 17.496715 for series X of 314 values