Long-range Dependence in the Returns and Volatility of the Brazilian Stock Market

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Abstract

This study provides empirical evidence of the long-range dependence in the returns and volatility of Brazilian Stock Market (BSM). We test for long memory in the daily returns and volatility series. The measures of long-term persistence employed are the modified rescaled range (R/S) statistic proposed by Lo (1991), the rescaled variance V/S statistic proposed by Giraitis *et al.* (2003), and the semiparametric estimator of Robinson (1995). Further analysis is conducted via FIGARCH model of Baillie *et al.* (1996). Significant long memory is conclusively demonstrated in the volatility measures, while there is a little evidence of long memory in the returns themselves. This evidence disputes the hypothesis of market efficiency and therefore implies fractal structure in the emerging stock market of Brazil. We conclude, that stock market dynamics in the biggest emerging market, even with its different institutions and information flows than the developed market, present similar return-generating process to the preponderance of studies employing other data. Our results should be useful to regulators, practitioners and derivative market participants, whose success depends on the ability to forecast stock price movements.

Keywords: Long Memory, R/S analysis, V/S analysis, Emerging Markets, Brazilian Stock Market

JEL classification: G1; G12; G14; G15

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1. Introduction

It is commonly observed that asset returns, whilst approximately uncorrelated, are temporally dependent. In particular, the autocorrelation functions of various volatility measures - squared, log-squared and absolute returns - decay at a very slow mean-reverting hyperbolic rate (see for example, Bollerslev and Wright (2000) and Ding, Granger and Engle (1993)). This feature is labelled a "long memory" or "long-range dependence". Long memory describes the correlation structure of a series at long lags. Such series are characterized by distinct but nonperiodic cyclical patterns. Mandelbrot (1977) characterizes long memory processes as having fractal dimensions. A widely accepted long memory time series model is the fractionally integrated ARFIMA (p, d, q) model. These models were introduced to economics and finance by Granger and Joyeux (1980) and Hosking (1981), and have the desired ability to match the slow decay of the autocorrelation functions. ARFIMA (p, d, q) models offered an alternative to ARIMA (p, d, q) process by not restricting the parameter d, to be limited to an integer value but rather allowing it to take on fractional values.

Because nonzero values of the fractional differencing parameter imply dependence between distant observations, considerable attention has been directed to the analysis of fractional dynamics in asset returns. Long-term dependencies have been found in the returns of a variety of assets classes. Cheung (1993), Cheung and Lai (1995), and Chou and Shih (1997) found evidence of a deterministic process in exchange rate changes. Helms, Kaen, and Rosenman (1984), Kao and Ma (1992), Eldridge, Bernhardt, and Mulvey (1993), Fang, Lai, and Lai (1994), and Corazza, Malliaris, and Nardelli (1997) found long-term dependence in index and commodity futures returns. Greene and Fielitz (1977), Lo (1991), and Nawrocki (1995) examined nonlinear regularities in U.S. equity market returns. Jacobsen (1996), Cheung, Lai, and Lai (1993) examined long-term dependence in developed European and Asian equity markets.

Despite the extensive research into the empirical and theoretical aspects of this relation in the well-developed financial markets, usually the U.S. markets, little is known about the information interaction in emerging securities markets. Emerging markets are typically much smaller, less liquid, and more volatile than well known world financial markets (Domowitz, Glen, and Madhavan (1998)). There is also more evidence that emerging markets may be less informationally efficient[1]. Further, the industrial organization found in emerging economies is

[1] This could be due to several factors such as poor-quality (low precision) information, high

often quite different from that in developed economies. All of these conditions and others may contribute to a different dynamics underlying returns and volatility in emerging stock markets.

Given the divergent conclusions of this research, further insights should be obtainable through an investigation of an alternative stock market returns, in particular, returns of an emerging market. The purpose of this study is to determine if long memory exists in the equity returns and volatility of the Brazilian Stock Market (BSM). We pay particular emphasis on the implications of long memory for market efficiency. According to the market efficiency hypothesis in its weak form, asset prices incorporate all relevant information, rendering asset returns unpredictable. The price of an asset determined in an efficient market should follow a martingale process in which each price change is unaffected by its predecessor and has no memory. If the return series exhibit long memory, they display significant autocorrelation between distant observations. Therefore, the series realizations are not independent over time and past returns can help predict futures returns, thus violating the market efficiency hypothesis. The second purpose is to examine the sensitivity of the findings to the choice of method of analysis. Our focus on the Brazilian Stock Market is appropriate for a number of reasons. First, Brazil is one of the countries in the Mercosur formed by the four Latin American countries and is becoming an increasingly important component of the regional and global economy. Its equity markets are integral segment of the South-American financial markets, and therefore, understanding the behavior of these markets is thus an important undertaking. Second, this market allows comparison of developed markets with maturing markets to determine if the returns-generating processes and presence or absence of chaos depends on the degree of market development. Third, the presence of long-memory in asset prices would provide evidence against the weak form of market efficiency and hence a potentially predictable component in the series dynamics. Fourth, the presence of fractal structure in equity prices may reflect fractal dynamic in the underlying economy which, in turn, would be of value in modelling business cycles. Fifth, as the volatility dynamic plays a very important role in derivative pricing, it may be beneficial to incorporate the long-term volatility structure in deriving pricing formulas. Indeed, Bollerslev and Mikkelsen (1996) presented results showing that it may be important to model the long memory volatility correctly when pricing contracts with long maturity, such as index options and futures.

Based on these results, we investigate the long-range dependence in the re-

trading costs, and/or less competition due to international investment barriers. For recent research on emerging markets and discussions of some of the differences between emerging and developed markets, see Errunza (1994); and Harvey (1995).

turns of the largest emerging markets in the world, namely Brazil. We investigate this property in the daily returns, from January 03, 1994 to May 17, 2002, with 2063 daily observations. We analyze the continuously compounded rate of return. A further application for long memory analysis lies in the dependence in the volatility of financial time series. We investigate this property in the market absolute returns, squared returns and modified log-squared returns. We use the modified rescaled range R/S statistic developed by Lo (1991), the rescaled variance V/S statistic developed by Giraitis *et al.* (2003) and the semiparametric Gaussian estimator of Robinson (1995). Besides testing for long memory, we model long-range dependence in volatility by using the FIGARCH (Fractionally Integrated GARCH) model of Baillie *et al.* (1996). Significant long memory is demonstrated in the volatility series, with a little evidence of dependence in the returns themsevles. This evidence is invariant to the method used in either testing or estimating the long memory components. Our findings for returns do not fall in line with those on other countries, while those regarding volatility are consistent consistent with the evidence reported by studies on developed markets. We conclude that the Brazilian market, even with its different institutions and information flows than the developed market, presents similar fractal market structure to the preponderance of studies employing other developed markets data. The implication of our results is that differences in institutions and information flows in Brazil are not that important enough to affect the valuation process of equity securities and produce similar results to those occurring in developed markets.

The paper is organized as follows. Section 2 provides an overview of the theoretical background and measures of volatility. Section 3 describes the tests and estimators employed. Section 4 presents the empirical results. Section 5 contains a summary of our findings and concluding remarks.

2. Long memory in volatility

Models with long memory in the volatility process have been proposed and found to match the autocorrelation functions of squared, log-squared and absolute asset returns. These include the fractionally integrated GARCH, or FIGARCH, model in Bollerslev and Mikkelsen (1996) and Baillie, Bollerslev and Mikkelsen (1996) and the fractionally integrated stochastic volatility model in Breidt, Crato and de Lima (1998). These models can imply that the autocorrelation functions of squared, log-squared and / or absolute returns have the same hyperbolic rate of decay as the volatility process.

To define a long memory model formally, a stationary stochastic process $\{X_t\}$ is called a long-memory process if there exists a real number H and a finite constant C such that the autocorrelation function $\rho(k)$ has the following rate of decay:

$$\rho(k) \sim Ck^{2H-2} \text{ as } k \longrightarrow \infty \tag{1}$$

The parameter H, called the Hurst exponent, may represent the long-memory property of the time series. A long-memory time series is also said fractionally integrated, where the fractional degree of integration d is related to the parameter H by the equality $d = H - 1/2$. If $H \in (1/2, 1)$, i.e., $d \in (0, 1/2)$, the series is *stationary* and said to have long-memory. If $H > 1$, i.e., $d > 1/2$, the series is nonstationary. If $H \in (0, 1/2)$, i.e., $d \in (-1/2, 0)$, the series is called antipersistent. Equivalently, a long-memory process can be characterized by the behavior of its spectrum $f(\lambda_j)$, estimated at the harmonic frequencies $\lambda_j = 2\pi j/n$, with $j = 1, ..., [n/2]$, near the zero frequency:

$$\lim_{\lambda_j \to 0^+} f(\lambda_j) = C\lambda_j^{-2d} \tag{2}$$

where C is a strictly positive constant and n is the sample size. The slow rate of decay of the autocorrelations of log-squared, squared and absolute returns motivates the construction of models with long memory in the volatility process.

3. Empirical methodology

3.1. The modified rescaled range analysis (R/S)

To detect for long-range dependence, Mandelbrot (1972) suggested the use of the range over standard deviation, R/S, which was *originally* developed by Hurst (1951). Lo (1991), however, showed that this statistic may be significantly biased when there is short-term dependence in the form of heteroskedasticity or autocorrelation, and suggested the use of the modified rescaled range statistic. To define the statistics formally, consider a sample of returns, $X_1, X_2, X_3, ..., X_n$ and let $\overline{X_n}$ denote the sample mean $(1/n)\sum_j X_j$. The rescaled range statistic, denoted by Q_n is defined as:

$$Q_n = 1/\widehat{\sigma}_x \left[Max \sum_{j=1}^{k} (X_j - \overline{X_n}) - Min \sum_{j=1}^{k} (X_j - \overline{X_n}) \right] \tag{3}$$

for $1 \leq k \leq n$, where $\widehat{\sigma}_x$ is the ML estimate of the standard deviation. The first term in Q_n is the maximum over k of the partial sums of the first k deviations of X_j from the sample mean. Since the sum of all n deviations of the

$X_j's$ from their mean is zero, this maximum is always nonnegative. The second term is the minimum over k of this same sequence of partial sums; it is always nonpositive. The difference of the two quantities, called the range, is therefore always nonnegative[2].

The difference between the traditional rescaled range and Lo's modified statistic is the denominator. The modified rescaled range statistic is:

$$Q_{n,q} = 1/\widehat{\sigma}_n(q)\left[Max\sum_{j=1}^{k}(X_j-\overline{X_n}) - Min\sum_{j=1}^{k}(X_j-\overline{X_n})\right] \tag{4}$$

for $1 \leq k \leq n$,where $\widehat{\sigma}_n^2(q) = \sigma_x^2 + 2\sum_{j=1}^{q} w_j(q)\widehat{\gamma}_j$,with $w_j(q) = 1 - j/(q+1)$, $q < n$ and $\widehat{\sigma}_x^2$ and $\widehat{\gamma}_j$ are the sample variance and autocovariance, respectively. The expression for Q_n differs from $Q_{n,q}$ only in its denominator, which is the square root of a consistent estimator of the partial sum's variance. If $\{X_t\}$ is subject to short-range dependence, the variance of the partial sum is not simply the sum of the variances of the individual terms, but also includes the autocovariances. That is the estimator $\widehat{\sigma}_n(q)$ involves not only sums of squared deviations of X_j, but also its weighted autocovariances up to lag q. The weights $w_j(q)$ are those suggested by Newey and West (1987) and always yield a positive $\widehat{\sigma}_n^2(q)$, an estimator of 2π times the spectral density function of X_t at frequency zero using a Bartlett window.

3.2. The rescaled variance V/S analysis

Equivalently, we can test for $I(0)$ against fractional alternatives by using the KPSS test of Kwiatkowski, Phillips, Schmidt, and Shin (1992), as Lee and Schmidt (1996) have shown that this test has a power equivalent to Lo's statistic against long-memory processes. The KPSS statistic for testing for long memory in a stationary sequence is given by:

$$KPSS(q) = \frac{1}{n^2\widehat{\sigma}^2(q)}\sum_{k=1}^{n} S_k^2 \tag{5}$$

where $\widehat{\sigma}^2(q)$ is the Newey and West (1987) heteroskedastic and autocorrelation consistent variance estimator of the centered observations $(X_j - \overline{X_n})$, for lag q and with $S_k = \sum_{j=1}^{k}(X_j - \overline{X}_n)$.

[2] Mandelbrot and Wallis (1969) demonstrated the superiority of R/S analysis in determining long-range dependence. They showed that the R/S statistic can detect long-range dependence in highly non-Gaussian time series with large skewness and kurtosis.

Giraitis *et al.* (2003) have proposed a centering of the KPSS statistic based on the partial sum of the deviations from the mean. They called it a rescaled variance test V/S as its expression given by:

$$V/S = \frac{1}{n^2 \widehat{\sigma}_n^2(q)} \left[\sum_{k=1}^{n} \left(\sum_{j=1}^{k} (X_j - \overline{X}_n \right)^2 - \frac{1}{n} \left(\sum_{k=1}^{n} \sum_{j=1}^{k} (X_j - \overline{X}_n \right)^2 \right] \tag{6}$$

can be equivalently rewritten as:

$$V/S = n^{-1} \frac{\widehat{V}\,(S_1, \ldots\ldots\ldots.S_n)}{\widehat{\sigma}_n^2(q)} \tag{7}$$

where S_k are again the partial sums of the observations. The V/S statistic is the sample variance of the series of partial sums $\{S_t\}_{t=1}^{n}$. The limiting distribution of this statistic is a Brownian bridge of which the distribution is linked to the Kolmogorov statistic. This statistic has uniformly higher power than the KPSS, and is less sensitive than the Lo statistic to the choice of the order q. For $2 \leq q \leq 10$, the V/S statistic can appropriately detect the presence of long-memory in the level series, although, like most tests and estimators, this test may wrongly detect the presence of long-memory in series with shifts in the levels.

3.3. Semiparametric gaussian estimator

To estimate a long memory time series model, one can rely on different proposed methods. However, since we are mainly interested in the long memory parameter d, this may be estimated by semiparameteric methods. Of these, the log-periodogram regression estimator is the most widely used. It was first proposed by Geweke and Porter-Hudak (1983). Robinson (1995) proposed an alternative semiparametric estimator of the long memory parameter d, which is asymptotically more efficient and the properties of which can be established under some mild conditions. Robinson (1995) estimator is based on the approximation of the spectrum of a long-memory process in the Whittle approximate maximum likelihood estimator. An estimator of the fractional degree of integration d is obtained by solving the minimization problem:

$$\{\widehat{C}, \widehat{d}\} = \underset{C,d}{\arg\min}\, L(C,d) = \frac{1}{m} \sum_{j=1}^{m} \left\{ \ln(C\lambda_j^{-2d}) + \frac{I(\lambda_j)}{C\lambda_j^{-2d}} \right\} \tag{8}$$

where $I(\lambda_j)$ is evaluated for a range of harmonic frequencies $\lambda_j = 2\pi j/n$, $j = 1, ..., m \ll [n/2]$ bounded by the bandwidth m,which increases with the sample size n but more slowly: the bandwidth m must satisfy $\frac{1}{m} + \frac{m}{n} \to 0$ as $n \to \infty$. If $m = n/2$, this estimator is the Gaussian estimator for the parametric model $f(\lambda) = C\lambda^{-2d}$. After eliminating C, the estimator $\widehat{d}$ is equal to:

$$\widehat{d} = \arg\min_{d} \left\{ \ln\left(\frac{1}{m}\sum_{j=1}^{m}\frac{I(\lambda_j)}{\lambda_j^{-2d}}\right) - \frac{2d}{m}\sum_{j=1}^{m}\ln(\lambda_j) \right\} \tag{9}$$

4. Data and empirical results

To analyze the Brazilian Stock Market (BSM), we use the daily index of the São Paulo Stock Exchange (BOVESPA). The period examined is from January 03, 1994 through May 17, 2002 with 2063 observations. In the case of a day following a nontrading day, the return is calculated using the closing price indices of the latest trading day and that day. We analyze the continuously compounded rate of return, $r_t = log(X_t/X_{t-1})$, where X_t denotes the stock index in day t. We also investigate the long-memory in the volatility by considering the series of absolute returns $|r_t|$, squared returns r_t^2 , and log-squared returns r_t^* as proxy of the volatility measures[3].

A problem often arises when dealing with log-squared returns; if the asset returns is very close to zero, then the log-squared transformation yields a large negative number. Such an observation can then greatly affect the results of subsequent data analysis. In the extreme case, if the asset return is equal to zero, then the log-squared transformation is not even defined. Fuller (1996) proposed a slight modification of the log-squared transformation, which does not converge to minus infinity as the argument converges to zero. This specifies that the transformed series of asset returns is:

$$r_t^* = \log(r_t^2 + \tau s^2) - \frac{\tau s^2}{(r_t^2 + \tau s^2)} \tag{10}$$

where s^2 is the sample variance of r_t and τ is a small constant. τ is set to 0.02, following Fuller (1996).

Table 1 summarizes the statistical properties of the returns: we show the first four moments, the autocorrelation coefficient at lag one and the Ljung and Box test statistic for autocorrelation in returns and squared returns. First, the higher variability of the Brazilian stock market returns is visible. Considering the

[3]There is little theoretical reason to prefer one volatility measure over any of the others. Lobato and Savin (1998) use squared returns, Granger and Ding (1996) use absolute returns, Breidt, Crato, and de Lima (1998) use log-squared returns and Bollerslev and Wright (2000) use all three of these volatility measures.

autocorrelation of returns, at lag one the BOVESPA has a value of 0.080 and is significant at the 5% level. In the table, we further observe two stylized facts for return series which has universal validity, as documented in the survey by Pagan (1996). The first stylized fact is nonnormality of the unconditional distribution of returns in the form of leptokurtosis. This phenomena has been termed fat tails. The second stylized fact is that the volatility of returns is time-varying. This dependence is indicated by the significant Ljung-Box Q(20) test statistics showing strong autocorrelation in squared returns. Thus, this time-dependence in volatility shows that a specification which omits the dynamics in variance neglects an important characteristic of the time series.

Table 1 also includes the implementation of KPSS tests proposed by Kwiatkowski *et al.* (1992) for the null hypothesis of $I(0)$ against long-memory alternatives. We consider two tests, denoted by Const and Trend based on a regression on a constant, and on a constant and time trend, respectively. As the table shows, the trend-stationarity null hypothesis is strongly rejected for return series. As a result, the return series cannot be characterized as $I(0)$ processes, which suggests that a fractionally differenced process may be an appropriate representation for these series.

In Table 2, we report the results from the R/S statistic. R/S is extremely sensitive to the order of truncation q and there is no statistical criteria for choosing q in the framework of this statistic. Andrews (1991) rule gives mixed results. If q is too small, this estimator does not account for the autocorrelation of the process, while if q is too large, it accounts for any form of autocorrelation and the power of this test tends to its size. Since there is no data driven guidance for the choice of this parameter, we consider different values for $q = 0, 2, 4, 6, 8, 10$ and 15. At the 5% significance level, the null hypothesis of a short-memory process is rejected if the modified R/S statistic does not fall within the confidence interval [0.809, 1.862].

For returns, the null hypothesis of short-memory is not rejected at any lag order. However, for volatility measures, the null hypothesis of short-memory is rejected for all lag orders, except for $q = 10$ and 15 for those of absolute and squared returns. The result illustrates the issue of the choice of the parameter q. In the case of absolute and squared returns, for $q = 0, 2, 4, 6$ and 8, we reject the null hypothesis of no long-memory. However, when $q = 10$, and 15, this null hypothesis is accepted, as the power of this test is too low for these levels of truncation orders. The results for the modified squared returns differ very much from those of the other two measures of volatility. In all cases, the null hypothesis of no long memory is rejected at the 5% significance level. This indicates that the BSM volatility measures are not IID and that certain patterns occur too frequently than otherwise be the case if the series were truly random.

In Table 3, we report the V/S statistic results. We consider the same truncation lag of q. There is again no evidence of long memory in the returns series, but strong and robust evidence of long range dependence in the volatility series. In only one case, where $q = 15$, the null hypothesis can not be rejected for squared returns. The presence of long memory is similar in value across all volatility measures and for each lag order. Our results fall in line with those reported by the R/S analysis. The combined evidence based on R/S and V/S statistics is a good representation of the data generating process, and suggests that a fractionally differenced process is the appropriate representation for these series.

To complement R/S and V/S analysis, we apply Robinson (1995) estimator to returns and volatility measures, using the following bandwidth parameters: $m = n/2, n/3, n/4, n/5, n/6$, and $n/7$. Our results are further supported by using the Geweke and Porter-Hudak (1983) estimator. The results from the two estimators are reported in Tables 4 and 5, respectively. Looking at returns, both estimators present similar results. We find no evidence of long memory at high periodogram points, in contrast to some some long memory when m is low. The existence of some long memory in the returns does not match the results obtained in Tables 2 and 3 (R/S and V/S analysis), since both tests were not able to reject the null hypothesis of short memory. Regarding volatility, significant and robust evidence of long memory can be found in the absolute returns, squared returns and modified log-squared returns. The estimated d values range between 0 and 0.5, which is the property of the fractionally integrated processes, in their ability to capture the long memory in returns and volatility when the fractional parameter d is in the range $(0 < d < 1/2)$, the ACF of such a model declines hyperbolically to zero, i.e., at a much slower rate of decay than the exponential decay of standard ARMA $(d = 0)$ process. The results are not too sensitive to the bandwidth, nor are they sensitive to the choice of volatility measures. However, we obtain lower estimates of d with the squared returns than with the other volatility measures, and the evidence of long memory is qualitatively the same across different choices of the periodogram points.

4.1. Further analysis: long memory via FIGARCH

Motivated by the presence of long-memory in the squared and absolute returns of various financial asset prices, Baillie, Bollerslev, and Mikkelsen (1996) proposed the fractionally integrated generalized autoregressive conditional heteroskedasticity (FIGARCH) model by combining the fractionally integrated process for the mean with the regular GARCH process for the conditional variance. This process implies a slow hyperbolic rate of decay for lagged squared innovations and persistent impulse response weights. A FIGARCH process exhibits the characteristic volatility effect captured by standard GARCH models, but with the difference that shocks to the error process die away at a slower, hypergoemetric rate rather than the short-term exponential decay typical of a short memory process. The FIGARCH $(1, d, 1)$ process is defined as:

$$r_t = \mu + \epsilon_t, \epsilon_t / \quad_t \sim \Delta(0, \sigma_t^2) \tag{11}$$

$$\sigma_t^2 = w + \{1 - [1 - \beta_1(L)]^{-1}(1 - \phi_1(L))[1 - L]^d\}\epsilon_t^2 \tag{12}$$

where μ is the unconditional mean of the process, $\quad_t$ is the information set at time t, Δ is the conditional distribution, L is the lag operator and w, β_1, ϕ_1 and d are parameters to be estimated with d being the fractional integration parameter. The FIGARCH $(1, d, 1)$ model nests the GARCH $(1, 1)$ model. For $d = 0$, then equation 14 reduces to the standard GARCH $(1, 1)$ model; and when $d = 1$, then equation 12 becomes the Integrated GARCH, or IGARCH $(1, 1)$, and implies complete persistence of the conditional variance to a shock in squared returns. As advocated by Baillie *et al.* (1996), the IGARCH process may be seen too restrictive as it implies infinite persistence of a volatility shock. Such a dynamics is not consistent with stylized facts. By contrast, for $0 < d < 1$, the FIGARCH model implies a long-memory behavior and slow rate of decay after a volatility shock.

As in the case of the GARCH model, the estimation of the FIGARCH model relies on the quasi maximum likelihood (QML) procedure. Following Bollerslev and Wooldrige (1992) one performs a correction of the standard errors of the estimates. Concerning the estimation procedure, two important points need to be made. The first one concerns the choice of the underlying distribution. As shown by Baillie *et al.* (1996) and Bollerslev and Wooldridge (1992), the QML estimates obtained with a Gaussian assumption behave relatively well. Nevertheless, as explained by Pagan (1996), a Student's-t distribution may be more appropriate to account for the leptokurticity characterizing the high frequency financial data. In this respect, we compare the results obtained with the Normal and with the Student's-t distributions. Therefore, the log-likelihood to be maximized becomes:

$$Ln(.) = T[log\{\Gamma\{(\nu + 1)/2\} - log\Gamma(\nu/2) - (1/2)\log \pi(\nu - 2)] - \tag{15}$$

$$-(1/2)\sum_{t=1}^{T}\{log(\sigma_t^2) + ((\nu + 1)[log(1 + \epsilon_t^2 \sigma_t^{-2}(\nu - 2)^{-1}]\},$$

where $\Gamma(.)$ is the gamma function and ν is the degrees of freedom parameter. A second point concerns the minimum number of observations required to estimate the FIGARCH model. This number is related to the order of the expansion of the fractional filter $(1-L)^d$. Because of the positive value of d, it is advisable to use a sufficiently high truncation lag order. In this respect, we chose a truncation order equal to 1000.

In order to assess the relevance of the FIGARCH specification, we also estimate a GARCH model. The estimation results are included in Table 6. Unsurprisingly, the Student's-t distribution is found to outperform the normal. Simple likelihood ratio tests point out that the degree of freedom ν needs to be included in the estimation procedure. As a whole, Table 6 suggests that the FIGARCH specification is supported by the data. Indeed, in all cases, the parameter d is highly significantly different both from 0 and 1, rejecting the validity of both the GARCH and the IGARCH specifications. Hence, there is strong support for the hyperbolic decay and persistence as opposed to the conventional exponential decay associated with the stable GARCH (1,1). Finally, a sequence of diagnostic statistics is provided and fail to detect any need to further complicate the model. These tests are skewness (b_3) and kurtosis (b_4) values as well as the Box-Pierce statistics of the residuals (Q(20)) and the squared residuals ($Q^2(20)$) at lag equal to 20. In general, the estimations carried out with assumed conditional Gaussian errors exhibit kurtosis, which tends to motivate further the use of a Student's-t distribution. As a whole, our MA(1) - FIGARCH $(1, d, 1)$ model and Student's-t distribution seems a satisfying representation to our data.

Significant evidence of long memory can be found in the volatility series; we find values of d different from zero and consistently significant. The evidence of long memory in the volatility is qualitatively the same across the different models. The estimated d values range between 0 and 0.5, a property of a process, in which its autocorrelation function declines hyperbolically to zero, i.e. at a much slower rate of decay than the exponential decay of standard ARMA ($d = 0$) process.

5. Conclusion

One of the important questions in studies of asset returns and volatility has been how long the effects of shocks persist. This is particularly important for emerging financial markets. This study attempts to investigate the long memory property in returns and volatility of the Brazilian Stock Market (BSM). Long memory is investigated via R/S statistic proposed by Lo (1991), V/S statistic developed by Giraitis *et al.* (2003) and the semiparametric Gaussian estimator of Robinson (1995). We also focus on the long memory in volatility by estimating the fractional parameter d within the FIGARCH model of Baillie *et al.* (1996). Significant long memory is found in absolute, squared, and modified log-squared returns, but not in the returns. These series exhibit significant long-range dependence, and similar to Ding *et al.* (1993) findings, the evidence of long memory is much stronger for absolute returns than for squared returns. In general, our results support the claim that the stock market returns in this emerging market has an underlying fractal structure, and disputes the hypothesis of market efficiency. Thus, we conclude that returns and volatility of Brazilian Stock Market (BSM), even with its different institutions and information flows than the developed market, present similar return-generating process and are *in tandem* with those patterns observed in the more mature stock markets of the developed countries. The implication is that differences in institutions and information flows in Brazil are not that important enough to affect the valuation process of equity prices and produce similar results to those occurring in developed markets.

Table 1: Statistical properties of returns

Mean	S.D.	Skewness	Kurtosis	$\widehat{\rho}_{r_t}(1)$	$Q_{r_t}(20)$	$\widehat{\rho}_{r_t^2}(1)$	$Q_{r_t^2}(20)$
0.0017	0.0292	0.591	12.25	0.080*	81.18*	0.195*	498*
KPSS Statistic			Critical values				
				0.1	0.05	0.01	
Const	1.125*			0.347	0.463	0.739	
Trend	0.229*			0.119	0.146	0.216	

Notes: * indicates significance at the 5% level of the null hypothesis of $I(0)$ against long-memory alternatives.

Table 2: Modified rescaled range (R/S) statistic for the returns, absolute, squared and modified log-squared returns

Lag order	R/S statistic			
q	r_t	$\lvert r_t \rvert$	r_t^2	r_t^*
0	1.835	2.813*	2.813*	4.686*
2	1.750	2.378*	2.378*	3.969*
4	1.759	2.128*	2.128*	3.466*
6	1.797	1.980*	1.980*	3.134*
8	1.795	1.869*	1.869*	2.891*
10	1.744	1.781	1.781	2.702*
15	1.655	1.626	1.626	2.361*

Notes: * indicates significance at the 5% level. At the 5% significance level, the null hypothesis of a short-memory process is rejected if the modified R/S statistic does not fall within the confidence interval [0.809, 1.862]. r_t, $|r_t|$, r_t^2, r_t^* represent returns, absolute, squared and modified log-squared returns, respectively.

Table 3: Variance Rescaled (V/S) statistic for the returns, absolute, squared and modified log-squared returns

Lag order	V/S statistic			
q	r_t	$\lvert r_t \rvert$	r_t^2	r_t^*
0	0.184	1.524*	0.553*	1.596*
2	0.167	0.947*	0.395*	1.145*
4	0.169	0.695*	0.316*	0.873*
6	0.176	0.561*	0.274*	0.714*
8	0.176	0.472*	0.244*	0.607*
10	0.166	0.411*	0.221*	0.531*
15	0.149	0.314*	0.184	0.405*

Notes: * indicates significance at the 5% level. The critical value is 0.1869 at 5% significance level. r_t, $|r_t|$, r_t^2, r_t^* represent returns, absolute, squared, and modified log-squared returns, respectively.

Table 4: Semiparametric estimates of d for returns, absolute, squared and modified log-squared returns, based on Robinson (1995)

Bandwidth	d estimates			
m	r_t	r_t^2	$\|r_t\|$	r_t^*
1031	0.015 $(0.020, 0.458)$	0.230* $(0.0207, 0.000)$	0.335* $(0.020, 0.000)$	0.231* $(0.02, 0.000)$
687	0.009 $(0.025, 0.705)$	0.235* $(0.0252, 0.000)$	0.351* $(0.025, 0.000)$	0.290* $(0.025, 0.000)$
515	0.0036 $(0.029, 0.899)$	0.251* $(0.029, 0.000)$	0.404* $(0.029, 0.000)$	0.317* $(0.029, 0.000)$
412	0.0846* $(0.0326, 0.009)$	0.268* $(0.032, 0.000)$	0.424* $(0.032, 0.000)$	0.321* $(0.032, 0.000)$
343	0.133* $(0.0358, 0.000)$	0.263* $(0.035, 0.000)$	0.449* $(0.035, 0.000)$	0.346* $(0.035, 0.000)$
294	0.217* $(0.038, 0.000)$	0.286* $(0.038, 0.000)$	0.498* $(0.038, 0.000)$	0.398* $(0.038, 0.000)$

Notes: * indicates significance at the 5% level. r_t, $|r_t|$, r_t^2, r_t^* represent returns, absolute, squared and modified log-squared returns, respectively. m represents the number of periodogram points. Standard errors and probabilities are provided in parentheses.

Table 5: Log periodogram estimates of d for returns, absolute returns, squared returns and modified log-squared returns based on Geweke and Porter-Hudak (1983)

Bandwidth	d estimates			
m	r_t	r_t^2	$\|r_t\|$	r_t^*
1031	0.0285 $(0.015, 0.066)$	0.258* $(0.0155, 0.000)$	0.365* $(0.015, 0.000)$	0.257* $(0.015, 0.000)$
687	0.0241 $(0.019, 0.205)$	0.249* $(0.0190, 0.000)$	0.382* $(0.019, 0.000)$	0.315* $(0.019, 0.000)$
515	0.0179 $(0.022, 0.415)$	0.263* $(0.022, 0.000)$	0.426* $(0.022, 0.000)$	0.347* $(0.022, 0.000)$
412	0.0694* $(0.024, (0.004)$	0.270* $(0.024, 0.000)$	0.436* $(0.024, 0.000)$	0.351* $(0.024, 0.000)$
343	0.104* $(0.0269, 0.000)$	0.273* $(0.026, 0.000)$	0.456* $(0.026, 0.000)$	0.372* $(0.026, 0.000)$
294	0.173* $(0.0291, 0.000)$	0.309* $(0.029, 0.000)$	0.498* $(0.029, 0.000)$	0.410* $(0.029, 0.000)$

Notes: * indicates significance at the 5% level. r_t, $|r_t|$, r_t^2, r_t^* represent returns, absolute, squared and modified log-squared returns, respectively. m represents the number of periodogram points. Standard errors and probabilities are provided in parentheses.

Table 6: Quasi maximum likelihood estimates of a FIGARCH model

	FIGARCH (N)	FIGARCH (St)	GARCH (N)	GARCH (St)
μ	0.183 (3.841)*	0.169 (3.694)*	0.182 (3.807)*	0.167 (3.621)*
θ_1	0.0729 (2.94)*	0.075 (3.159)*	0.075 (3.079)*	0.074 (3.122)*
ω	0.275 (3.391)*	0.164 (2.299)*	0.235 (4.287)*	0.132 (3.007)*
β_1	0.578 (4.435)*	0.629 (5.571)*	0.805 (35.53)*	0.858 (37.84)*
ϕ_1	0.128 (1.641)	0.147 (1.972)*	0.170 (8.181)*	0.128 (5.929)*
d	0.325 (5.270)*	0.348 (5.446)*	-	-
ν	-	7.767 (7.112)*	-	8.140 (6.714)*
Ln (L)	-4729.61	-4686.88	-4729.93	-4688.4
Q(20)	35.34	27.30	26.809	26.22
$Q^2(20)$	11.07	18.65	17.41	20.68
b_3	-0.043	-0.274	-0.262	-0.289
b_4	4.496	2.296	1.947	2.286

Notes: t-statistics of maximum likelihood estimates are in brackets. * indicates rejection at the 5% level. St and N refer, respectively, to estimations with the Student and the Normal distributions. Ln (L) is the value of the maximized log likelihood. The sample skewness and kurtosis refer to the standarized residuals. The Q(20) and $Q^2(20)$ statistics are the Ljung-Box test statistics for 20 degrees of freedom to test for serial correlation in the standarized residuals and squared standarized residuals.

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