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Possible causes of long-range dependence in the Brazilian stock market

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Abstract

While the presence of long-range dependence in the asset returns seems to be a stylized fact, the issue of arguing the possible causes of this phenomena is totally obscure. Trying to shed light in this problem, we investigate the possible sources of the long-range dependence phenomena in the Brazilian Stock Market. For this purpose, we employ a sample which comprises stocks traded in the Brazilian financial market (BOVESPA Index). The Hurst exponent here is considered as our measure of long-range dependence and it is evaluated by six different methods. We have found evidence of statistically significant rank correlation between specific variables of the Brazilian firms which subscribe stocks and the long-range dependence phenomena present in these stocks.

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1. Introduction

The presence of long-range dependence in asset returns has been intriguing academicians as well as financial market professionals for a long time. One of the

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first to consider the existence of long memory behavior in asset returns was Mandelbrot [1]. Since then, many others have supported Mandelbrot's results (for details see [2–7] and the references therein).

Actually, while the presence of long-range dependence in the asset returns seems to be a stylized fact, the issue of arguing the possible causes of this phenomena is totally obscure.[1] The question that usually arises is why we find evidence of long-range dependence for stocks of some firms and we do not find for stocks of other firms? This is an important question that to the best of our knowledge has not been addressed in the literature before. Therefore, due to the importance of the implications of the presence this phenomena in financial data,[2] this paper aims at shedding some light in this issue, so we investigate the possible causes of the long memory phenomena in the Brazilian Stock Market.

For this purpose, we employ some proxies for specific firm variables: capitalization measures (proxy for liquidity), dividends payments, return on equity (ROE) and financial leverage. The sample considered in this work comprises all stocks traded in the Brazilian financial market and the period of this research stems from January 1998 through November 2003. The data sampling employs daily closing prices for individual stocks. Additionally, the Hurst exponent is thought here as our measure of long-range dependence and to give some robustness to our results, we evaluate it by six different methods: R/S analysis, R/S analysis with shuffled data, R/S analysis with data aggregation, DFA, DFA with shuffled data and DFA with data aggregation. It is interesting to stress that we avoid here using the modification of the R/S method proposed by Lo [12] since this method has a strong preference for accepting the null hypothesis of no long-range dependence independently of whether long-range dependence is presented in the data or not (for details, see Refs. [13,14]). We present standard errors for Hurst exponents for each one these methods, which can be used to test for the null of $H = 0.5$ (efficient market).

This paper is organized as follows. The methods used to evaluate the Hurst exponent are introduced in Section 2. The proxies for specific firm variables are presented in Section 3. In Section 4, the data used in this work is presented. In Section 5, the results are exposed. Finally, Section 6 presents some conclusions of this work.

[1]Cajueiro and Tabak [8] present some evidence suggesting that liquidity and market restrictions play a role in explaining empirical results from testing for long-range dependence.

[2]The evidence of long-range memory in financial data causes several drawbacks in modern finance: (1) the optimal consumption and portfolio decisions may become extremely sensitive to the investment horizon [9]; (2) the methods used to price financial derivatives based on martingale models (the most common models, e.g. the Black–Scholes model [10]) are not useful anymore; (3) since the usual tests based on the Capital Asset Pricing Model and Arbitrage Pricing Theory [11] do not take into account long-range dependence, they cannot be applied to series that present such behavior. Moreover, if such long-range persistence is presented in the returns of the financial assets, the random walk hypothesis is not valid anymore and neither does the market efficiency hypothesis.

2. Methods used to evaluate the Hurst exponent

2.1. The classical R/S analysis

Due to its simplicity, the most popular methodology to measure long-range dependence is the so-called R/S analysis [15,16]. Its measure of long-range dependence is based on the evaluation of the Hurst's exponent of stationary time series (in finance, generally it is used the log returns to evaluate the Hurst's exponent H). More explicitly, let $X(t)$ be the price of a stock on a time t and $r(t)$ be the logarithmic return denoted by $r(t) = \ln(X(t+1)/X(t))$. The R/S statistic is the range of partial sums of deviations of times series from its mean, rescaled by its standard deviation. So, consider a sample of continuously compounded asset returns $\{r(1), r(2), \ldots, r(\tau)\}$ and let $\bar{r}(\tau)$ denote the sample mean $(1/\tau)\sum_\tau r(\tau)$ where τ is the time span considered. Then the R/S statistic is given by

$$(R/S)_\tau \equiv \frac{1}{s_\tau}\left[\max_{1\leqslant t\leqslant\tau}\sum_{k=1}^{t}(r(k)-\bar{r}(\tau)) - \min_{1\leqslant t\leqslant\tau}\sum_{k=1}^{t}(r(k)-\bar{r}(\tau))\right], \tag{1}$$

where s_τ is the usual standard deviation estimator

$$s_\tau \equiv \left[\frac{1}{\tau}\sum_t (r(t)-\bar{r}(\tau))^2\right]^{1/2}. \tag{2}$$

Hurst [15] found that the rescaled range, R/S, for many records in time is very well described by the following empirical relation:

$$(R/S)_\tau = (\tau/2)^H . \tag{3}$$

So, the Hurst's exponent may be evaluated by plotting the data $(R/S)_\tau$ versus τ in a log–log plot and measuring the slope of the straight line.

The main drawback of the R/S analysis is that its measure of long-range dependence is affected by short-range dependence that may be presented in the financial data. Therefore, in this work to avoid this problem, we consider two extensions of the R/S analysis that can remove this extra short-range dependence: (1) We apply the R/S analysis to shuffled data in blocks of size 5, i.e., we pick a random permutation of the data series within each block of size 5 and apply the R/S analysis to this shuffled data. The effect of random permutations in these small blocks is to destroy any particular structure of autocorrelation within these blocks (shuffled data was used, for instance, in the context of long-range dependence in [17]). (2) We apply the R/S analysis to aggregated data, i.e., we take the sample mean of non-overlapping blocks of size 5 and we apply the R/S analysis to this manipulated data (aggregated data was used, for instance, in the context of long-range dependence in [18]). One should note that aggregating the data, the resulting data series becomes more Gaussian and short-range dependence tends to become insignificant.

2.2. The detrended fluctuation analysis

The detrended fluctuation analysis (DFA) was developed independently in Refs. [19,20] and provides an alternative for the determination of the Hurst exponent. Different from the R/S analysis, the DFA evaluates the Hurst's exponent using the integrated time series of logarithmic returns.

Let $Y(t)$ be the integrated time series of logarithm returns, i.e., $Y(t) = \log(X(t))$. So, one considers the τ-neighborhood around each point $Y(t)$ of the time series. The local trend in each τ-size box is approximated by a polynomial[3] of order m, namely $Z(t)$).

Then, one evaluates the local roughness, namely

$$w^2(Y, \tau) = \frac{1}{\tau} \sum_{t \in \tau} (Y(t) - Z(t))^2 . \tag{4}$$

It is easy to show [19] that

$$\langle w^2(\tau) \rangle \sim \tau^{2H} . \tag{5}$$

3. Proxies for firms variables

Our proxies for specific firm variables are market capitalization measures (proxy for liquidity), dividend payments, ROE and financial leverage. To the best of our knowledge, this is the first paper that relates these variables with long-range dependence measures. We provide some rationale on why these variables should be linked together. However, we do not propose a formal model explaining their interactions.[4]

The market capitalization is the total market value of the individual companies that entered in our sample. In general, firms with a high market capitalization tend to have a high traded value when compared to their lower market capitalization counterparts. Therefore, we should expect that this variable would be negatively related to long-range dependence, as increasing trading activity should render markets more efficient.[5]

If firms pay dividends on a regular basis and their earnings are somewhat predictable, then stock prices would reflect that and there could be long-range predictability. However, if dividends payments are erratic and unpredictable then there should be no relationship between our long-range dependence measures and

[3]This polynomial of order m is usually a first-order polynomial, i.e., a straight line where the parameters are determined by a least-square fitting.

[4]Formal models dealing with this issue are particularly welcome.

[5]Tabak [21] has shown by means of a "rolling-sample" approach and using short-range dependence measures (in particular, the variance ratio statistics was used), which is the case for Brazil. Basically, the author has shown that with the opening of the domestic equity market capital portfolio, inflows have increased substantially domestic liquidity and the market has converged towards efficiency in Brazil. However, this paper only analyzes the market index using short-range dependence measures.

this variable. Considerable evidence exists to support the hypothesis that the payment of dividends provides information that helps investors and analysts value the firm.[6]

Another variable is the ROE, which reflects the gains that stockholders are making for staying long in a specific stock. Greater ROE means that these stocks are more desirable and should attract more investments, and perhaps should be associated with increases in liquidity. Therefore, we would expect a negative relationship between this variable and long-range dependence measures.

Financial leverage ratios could be negatively associated with long-range dependence measures, as increases in the proportion of debt for firms would tend to increase risks and the cost of capital of firms. Therefore, these firms would have to search for projects with higher net present value or high rates of return, which have more risk, decreasing predictability of their net results. We expect a positive relationship between long-range dependence measures and this variable.

4. The data

The data used here comprises of all stocks that belong to the Brazilian São Paulo Stock Exchange Index (IBOVESPA) and the period of this research stems from January 1998 through November 2003. This study employs daily closing prices for individual Brazilian stocks. In the Brazilian equity market, these stocks are the most liquid ones, since the condition to enter in the index is, they must have liquidity. Actually, stocks are considered liquid depending on both trading values and number of days that they are traded. The median number of observations is 1387.

Therefore, with our sampling approach we are controlling very low liquidity which sometimes characterizes many stocks for emerging markets.

On the other hand, another important issue is that, in general it is very difficult to build strategies for an index itself and finding long memory in the index may suggest that one could build models to forecast the index and trade on these forecasts. Due to difficulties inherent in building strategies for aggregate indices (for example, to deal with a large number of stocks) using this data it is possible to ascertain whether individual stocks possess long memory as well. Thus, one could use these results to build trading models for stocks which possess long memory.

5. Empirical Results

Table 1 presents the results found for the Hurst exponent using the six methods already introduced for Brazilian stocks and the average of these methods.

The first thing that is worth noting is that although the mean and median of the last column of Table 1 are, respectively, 0.539 and 0.54 which would suggest an efficient market, average Hurst exponents range from 0.45 to 0.65, which are

[6] See [22] and [23] which predicts a positive association between dividends and stock prices.

Table 1
Hurst exponent evaluated by six different methods and its average. The first and second column for each method present the Hurst exponent and its respective standard error

Stocks	R/S		R/S with shuffling		R/S with aggregation		DFA		DFA with shuffling		DFA with aggregation		Average
ITSA4	0.57	0.02	0.57	0.02	0.60	0.03	0.53	0.01	0.52	0.01	0.49	0.02	0.55
KLBN4	0.62	0.02	0.62	0.02	0.69	0.02	0.56	0.02	0.56	0.01	0.73	0.01	0.63
LIGH3	0.59	0.01	0.60	0.02	0.65	0.03	0.55	0.01	0.56	0.01	0.55	0.01	0.58
PLIM4	0.65	0.03	0.65	0.03	0.70	0.03	0.61	0.01	0.60	0.01	0.70	0.02	0.65
PETR3	0.56	0.02	0.57	0.02	0.59	0.02	0.56	0.01	0.56	0.01	0.53	0.02	0.56
PETR4	0.52	0.02	0.53	0.02	0.55	0.03	0.52	0.01	0.53	0.01	0.45	0.02	0.52
SBSP3	0.60	0.02	0.59	0.02	0.61	0.02	0.57	0.01	0.58	0.01	0.44	0.02	0.57
CSNA3	0.56	0.02	0.58	0.02	0.63	0.03	0.55	0.01	0.55	0.01	0.53	0.01	0.57
CSTB4	0.59	0.03	0.60	0.02	0.66	0.02	0.55	0.00	0.55	0.00	0.52	0.01	0.58
CRUZ3	0.48	0.02	0.45	0.02	0.51	0.03	0.47	0.00	0.46	0.00	0.45	0.01	0.47
TCOC4	0.57	0.02	0.57	0.01	0.60	0.02	0.46	0.01	0.46	0.00	0.57	0.02	0.54
TCSL3	0.50	0.03	0.50	0.03	0.54	0.04	0.45	0.01	0.45	0.01	0.41	0.01	0.48
TCSL4	0.54	0.03	0.53	0.03	0.57	0.04	0.47	0.01	0.47	0.01	0.46	0.01	0.51
TLCP4	0.57	0.02	0.54	0.02	0.59	0.03	0.50	0.00	0.51	0.00	0.52	0.01	0.54
TNEP4	0.51	0.02	0.51	0.02	0.54	0.04	0.46	0.01	0.48	0.01	0.45	0.01	0.49
TNLP4	0.53	0.02	0.55	0.02	0.57	0.03	0.46	0.01	0.48	0.01	0.45	0.01	0.51
TNLP3	0.48	0.03	0.48	0.03	0.54	0.04	0.45	0.00	0.45	0.01	0.49	0.02	0.48
TMAR5	0.55	0.01	0.55	0.02	0.59	0.02	0.49	0.01	0.50	0.01	0.41	0.01	0.51
TMCP4	0.53	0.02	0.54	0.02	0.57	0.03	0.47	0.01	0.47	0.01	0.47	0.02	0.51
TSPP4	0.55	0.03	0.56	0.04	0.54	0.04	0.53	0.01	0.53	0.01	0.54	0.02	0.54
TLPP4	0.55	0.01	0.53	0.02	0.58	0.03	0.52	0.01	0.52	0.01	0.50	0.02	0.53
TBLE3	0.54	0.02	0.53	0.02	0.58	0.04	0.51	0.01	0.49	0.01	0.44	0.02	0.51
USIM5	0.62	0.02	0.63	0.01	0.66	0.02	0.55	0.01	0.56	0.01	0.56	0.01	0.60
VCPA4	0.46	0.03	0.48	0.02	0.53	0.04	0.43	0.01	0.43	0.01	0.37	0.02	0.45

VALE3	0.55	0.01	0.56	0.01	0.62	0.02	0.50	0.00	0.50	0.01	0.44	0.01	0.53
VALE5	0.51	0.02	0.51	0.02	0.56	0.02	0.49	0.01	0.49	0.01	0.41	0.01	0.50
ACES4	0.61	0.02	0.61	0.02	0.66	0.02	0.55	0.01	0.55	0.01	0.62	0.01	0.60
AMBV4	0.52	0.02	0.54	0.02	0.59	0.02	0.48	0.01	0.48	0.01	0.48	0.02	0.51
ARCZ6	0.55	0.02	0.53	0.02	0.59	0.02	0.52	0.01	0.51	0.01	0.52	0.01	0.54
ITAU4	0.51	0.02	0.51	0.01	0.52	0.03	0.46	0.01	0.47	0.01	0.43	0.02	0.48
BBDC4	0.54	0.02	0.54	0.02	0.56	0.03	0.52	0.00	0.52	0.01	0.44	0.01	0.52
BRAP4	0.55	0.03	0.54	0.03	0.67	0.03	0.53	0.02	0.52	0.02	0.64	0.02	0.58
BBAS3	0.54	0.02	0.53	0.02	0.57	0.03	0.51	0.00	0.49	0.00	0.41	0.02	0.51
BRTP4	0.52	0.02	0.51	0.02	0.56	0.03	0.43	0.01	0.45	0.01	0.39	0.01	0.48
BRTP3	0.53	0.02	0.50	0.02	0.56	0.03	0.45	0.01	0.45	0.01	0.44	0.01	0.49
BRTO4	0.57	0.01	0.55	0.01	0.59	0.02	0.51	0.01	0.51	0.01	0.50	0.01	0.54
BRKM5	0.62	0.02	0.62	0.02	0.65	0.02	0.60	0.01	0.60	0.01	0.65	0.01	0.62
CLSC6	0.56	0.02	0.57	0.01	0.62	0.02	0.52	0.01	0.52	0.01	0.38	0.02	0.53
CMIG3	0.51	0.02	0.49	0.02	0.57	0.03	0.46	0.00	0.46	0.00	0.36	0.02	0.47
CMIG4	0.53	0.02	0.53	0.02	0.57	0.03	0.46	0.01	0.48	0.01	0.35	0.02	0.49
CESP4	0.54	0.03	0.54	0.03	0.59	0.03	0.60	0.00	0.60	0.00	0.53	0.02	0.57
TRPL4	0.58	0.01	0.57	0.01	0.64	0.02	0.51	0.01	0.51	0.01	0.41	0.02	0.54
CGAS5	0.60	0.02	0.60	0.02	0.63	0.02	0.54	0.00	0.54	0.00	0.61	0.01	0.59
CPLE6	0.54	0.01	0.53	0.01	0.57	0.02	0.48	0.01	0.50	0.01	0.40	0.03	0.50
CRTP5	0.59	0.01	0.59	0.01	0.65	0.02	0.60	0.00	0.61	0.00	0.57	0.01	0.60
ELET3	0.52	0.02	0.54	0.02	0.57	0.03	0.49	0.01	0.51	0.01	0.37	0.02	0.50
ELET6	0.52	0.02	0.54	0.02	0.58	0.03	0.49	0.01	0.49	0.01	0.36	0.02	0.50
ELPL4	0.53	0.02	0.54	0.02	0.57	0.02	0.50	0.01	0.50	0.01	0.51	0.02	0.53
EMBR3	0.60	0.01	0.60	0.01	0.65	0.02	0.58	0.01	0.58	0.01	0.59	0.02	0.60
EMBR4	0.60	0.02	0.60	0.02	0.64	0.02	0.57	0.01	0.58	0.01	0.66	0.01	0.61
EBTP3	0.52	0.02	0.54	0.02	0.60	0.02	0.53	0.01	0.54	0.01	0.63	0.01	0.56
EBTP4	0.56	0.03	0.56	0.03	0.59	0.03	0.57	0.01	0.57	0.01	0.62	0.01	0.58
GGBR4	0.59	0.02	0.59	0.02	0.64	0.02	0.53	0.01	0.54	0.00	0.53	0.01	0.57
PTIP4	0.55	0.02	0.54	0.02	0.61	0.02	0.49	0.01	0.47	0.01	0.55	0.02	0.53

substantially low and high and distorts aggregate results. These results illustrate the importance of studying individual stocks as well as aggregate indices (as most works do) since aggregation may lead to erroneous conclusions regarding the overall efficiency of a particular equity market.

On the other hand, from Table 1 it is easy to see that stocks that have a huge base of shareholders such as Vale do Rio Doce (VALE5) and Eletrobras (ELET3 and ELET6) have an average Hurst exponent equal to 0.5, indicating high efficiency. It is worth noting that we present standard errors (for each method used to estimate Hurst exponents) right after presenting Hurst exponents. For example, for the ELET3 stock, the estimated Hurst exponent (using the R/S measure) is 0.52, while the standard errors is given by 0.02. Therefore, a T-statistic for this coefficient would be given by $T_{stat} = (0.52 - 0.50)/0.02 = 1$, which suggests that the null coefficient is equal to 0.5 cannot be rejected.

These stocks that possess a Hurst exponent close to 0.5 are amongst the most liquid in the sample. Therefore, using this assertion as a motivation, we test for rank correlation between variables such as liquidity and proxies for financial leverage and profitability to understand what drives the results obtained in Table 1.

Using the last column of Table 1 and non-parametric correlation test (for details, see appendix), we infer whether high (or low) Hurst exponents have a significant relationship with high (or low) market capitalization (which, is a proxy for liquidity for these stocks), high (or low) stream of dividends payments, high (or low) return on equity and high (or low) financial leverage ratios.

The empirical results are that the rank correlation coefficient[7] between the average long-range dependence measures and ROE is -33.37%; between the average long-range dependence measures and financial leverage (ratio of debt to total assets) is 31.12% and between the average long-range dependence measures and market capitalization is -31.14%. These correlations are statistically significant at the 5% level and they have the expected signs. However, the rank correlation between the average long-range dependence measures and dividends payments is very low, approximately 9.99% and is not statistically significant, suggesting that dividends might be themselves hard to predict, which seems reasonable for such a highly volatile market as the Brazilian equity market.

Therefore, our measures suggest that firm-specific variables can explain, at least partially, the long-range dependence phenomena (for details, see Table 2).

Actually, if traders and market practitioners price stocks using fundamentals we would expect these variables to have a significant high correlation with long-range dependence measures. However, the small correlations (although significant) suggest that agents do not employ solely fundamentals to price stocks.

[7]A rank correlation coefficient is a coefficient of correlation between two random variables that is based on the ranks of the measurements and not the actual values. Therefore, if this coefficient is statistically significant at the 5% level we can infer that high (and lows) long-range dependence measures are statistically associated to high (and low) market microstructure variables for the stocks that comprise our sample.

Table 2
Rank correlation for the firm variables

	H (%)	ROE (%)	M. Cap. (%)	Dividends (%)	Debt/assets (%)
H	100.00	−30.38*	−31.41*	9.99	31.11*
ROE	−30.38*	100.00	4.41	−23.68	−35.87*
M. Cap.	−31.41*	4.41	100.00	38.92*	13.14
Dividends	9.99	−23.68	38.92*	100.00	38.51*
Debt/assets	31.11*	−35.87*	13.14	38.51*	100.00

*Significant at the 5% level.

The component of long memory which is not explained by these specific firm variables may be probably explained by other sources such as technical analysis trading [24]. It has been shown that speculative behavior can induce long-range dependence. Our paper provides additional insight on this matter. Since correlations between Hurst exponents and fundamentals are low, we would have that long-range dependence may be generated by trading mechanisms that do not employ information on fundamentals but on other variables such as technical analysis.

Another explanation for the results found in this paper are the possibility of presence of bubbles in the financial market [25]. Bubbles are generated when stocks prices are driven not only by fundamental but also by other variables such as speculative behavior. Suppose that a trader finds that the fundamentals of a particular company are bad, but he thinks that the stock will have its price increased in the next days. The trader could stay long in the stock and buy more stocks as long as he believes that prices would go up. Obviously, this trader knows that eventually he will have to sell the stock, but his behavior in the short run may induce prices to behave differently from what one would expect from fundamentals. In general, a financial market can be divided in three groups of traders: the so-called regular traders who choose investments based on the information of firms fundamental variables, the so-called technical traders who trade stocks based on the information of technical indicators and traders driven by herding behavior who, in general, cause bubbles in financial stocks. So, the information used by these three groups should explain the overall long-range dependence phenomena. Since, we are concerned here only with part of this information, we are able to explain only part of this phenomena.

6. Final conclusions

In this paper, we have studied possible sources of the long-range dependence presented in returns of the Brazilian financial stocks. Using the Hurst exponent as our measure of long-range dependence and specific variables of the Brazilian firms which subscribe stocks, we have found evidence that firm specific variables can explain, at least partially, long-range dependence measures. We claim that this paper represents an advance in our understanding of such phenomena.

Our findings suggest that prices are not solely driven by fundamentals but also by other market characteristics. Speculative behavior (for example, technical analysis) and speculative bubbles in stock markets have important roles in the determination of prices. More research is needed in order to enhance our knowledge of how these mechanisms affect prices and generate long-range dependence.

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Appendix. The rank correlation test

The rank correlation test is a non-parametric technique used to test the degree of relationship between two variables when the data is disposed in ranks.

To measure the rank correlation is used the so-called Spearman rank correlation coefficient r_S which is calculated by using the ranks of the paired measurements of two variables X and Y and the usual correlation definition. Thus,

$$r_S = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} , \tag{6}$$

where $S_{xy} = \sum_{i=1}^{n} x_i y_i$, $S_{xx} = \sum_{i=1}^{n} x_i^2$, $S_{yy} = \sum_{i=1}^{n} y_i^2$, $x_i = X_i - \overline{X}$ and $y_i = Y_i - \overline{Y}$.

It can be proved that r_S lies between -1 and $+1$ and $|r_S|$ near 1 means that there is a high correlation between the two variables X and Y.

This coefficient may be employed as a test statistic to test a hypothesis of no association between two populations. We assume that the n pairs of observations (x_i, y_i) have been randomly selected and, therefore, the absence of any association between the populations implies a random assignment of the n ranks within each sample. Each random assignment (for the two samples) represents a sample point associated with the experiment, and a value of r_S can be calculated for each. It is possible to calculate the probability that r_S assumes a large absolute value due solely to chance and thereby suggests an association between populations when none exists.

The rejection region for a two tailed test includes values of r_S near $+1$ and near -1. If the alternative is that the correlation between X and Y is negative, we reject the hypothesis of no association between these two populations for values near -1. Similarly, if the alternative is that the correlation between X and Y is positive, we reject the hypothesis of no association between these two populations for values near $+1$. If size of the samples is bigger than 10, the critical values for this statistic may be evaluated by

$$t = \frac{r_S}{\sqrt{\frac{1-r_S}{n-2}}} \tag{7}$$

which is a t-distribution with $n-2$ degrees of freedom.

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