Are There Long Cycles in Common Stock Returns?

KURSAT AYDOGAN
Bilkent University
Ankara, Turkey

G. GEOFFREY BOOTH
Louisiana State University
Baton Rouge, Louisiana, U.S.A.

I. Introduction

An enormous amount of empirical work has been directed toward understanding the nature of stock price movements. Early studies, such as those by Alexander [2; 3], Fama [10], and Fama and Blume [11], support the contention that stock price changes follow a random walk. Thus cycles that appear to exist are simply a statistical artifact. More recent efforts, namely Young [28], Greene and Fielitz [12], Praetz [24], and Renshaw [25], however, provide some evidence against this hypothesis and suggest that stock price changes follow some type of a cyclic time path.[1] Explanations for these contradictory findings rely on the way that information is disseminated to, processed by, and acted on by market participants. To date, three basic information-based scenarios have appeared in the literature: (1) not all information is of equal importance, (2) not all participants have the same information, and (3) not all participants are equal in their ability to process information.

This latter scenario is the most recent and has been used by Heiner [13] to develop a theory of predictable behavior by constructing a competence-difficulty (C-D) gap construct. Kaen and Rosenman [16] extend Heiner's [13] work to provide an explanation for the presence of nonperiodic cycles in asset prices, especially the prices (and returns) of common stock. They maintain that, according to Heiner's theory, market participants differ in their ability in making the correct buy and sell decisions under uncertainty. In financial markets, the spread between the participants' competence in conjunction with the complexity of the information, i.e., the C-D gap, results in price changes in the same direction. If new contradictory information is of substantial import, directional change occurs. Since the arrival of new information is posited to a random event, the resulting periods of similar price change behavior appear to be nonperiodic cycles. Empirically, Kaen and Rosenman [16] support their contention by citing several studies that have used rescaled

1. Respectively, these studies employ variance time function analysis, rescaled range analysis, spectral analysis, and trading rules. Conceptually, in this application all approaches but rescaled range are somewhat suspect: variance time and spectral analysis because of the characteristics of the marginal distribution (e.g., moments may not be defined) and, to a lesser extent, trading rules because the trade indicated by ex post rules may not be able to be accomplished ex ante. Rescaled range analysis is also subject to implementation problems; these will be discussed later in the paper.

range (R/S) analysis to measure nonperiodic cycles in economic time series; special emphasis is given to the Greene and Fielitz [12] study of stock returns and the rescaled range technique itself.

As a statistical technique, R/S analysis has come under severe criticism since its introduction by Hurst [15], much of which surfaced subsequent to the Greene and Fielitz [12] study. The focal point of the analysis is the Hurst coefficient, which is a proportionality factor with an expected asymptotic value of one-half. Empirical observation of a Hurst value exceeding one-half is evidence of the Hurst phenomenon. Two radically different explanations have been offered to account for this observation. One, which was comprehensively introduced by Mandelbrot and Wallis [20] and later extended in an economic context by Mandelbrot [17; 18], maintains that the true underlying stochastic process for price change series is fractional Brownian motion. For this process, the existence of the Hurst phenomenon is interpreted as an indication of long-term dependence in which observations in the past influence observations far in the future. The other explanation regards the Hurst phenomenon as an empirical anomaly caused by certain features in the series such as autocorrelation and trend [4; 26] and nonhomogeneities [6; 22]. Complex permutations of these intrinsic characteristics may either delay or even preclude the Hurst coefficient from ever obtaining its theoretical asymptotic value of one-half.[2] The impact of all preasymptotic behavior, except that relating to mean nonhomogeneity (a shifting mean), may be adequately mitigated if the time series is long enough to enable the estimated Hurst coefficient to converge to its theoretical limit. Unfortunately, except for some investigations into the issue of bias, little research has been done concerning optimal record length. Extant empirical studies, including the one by Greene and Fielitz [12], appear either to use all the data available or to employ a sample that is considered extremely long (e.g., 1000) by traditional econometric rather than R/S standards. Nevertheless, the existence of preasymptotic behavior, whatever its origin, suggests that strong reliance on the value of the Hurst coefficient to draw conclusions concerning nonperiodic cycles may be unwarranted.

Therefore, because of the potential importance of the Kaen and Rosenman supposition and of the implications of the presence of cycles in stock price movements in general, the purpose of this study is to re-evaluate and extend the Greene and Fielitz [12] study by explicitly addressing the issue of preasymptotic behavior and by using a longer (chronological not observational) data set.[3] The findings indicate that although the Hurst coefficients are somewhat similar to those obtained by Greene and Fielitz [12], as a group they are not significantly different from white noise when those estimates suspected of displaying preasymptotic behavior are eliminated. It is important to recognize that this finding does not deny the C-D gap hypothesis as postulated by Kaen and Rosenman [16], but it does suggest that additional supporting evidence should precede its wholehearted acceptance.

II. The Data

Two hundred randomly selected stocks are analyzed for the 18.5 year period beginning July 1962 and ending December 1980. For each stock, 965 weekly rates of return are computed from the

2. Mandelbrot and his compatriots acknowledge the existence of preasymptotic behavior, but they believe that its impact dies out very quickly and can thus be statistically avoided. Their technique is subsequently discussed.

3. The usefulness of this re-examination is further underscored by noting that the C-D gap hypothesis is surfacing in other market behavior literature. For example, Biasco [5] uses it as part of his explanation for the presence of cycles in foreign exchange rates.

CRSP daily stock return file. Weekly rates of return for the CRSP value-weighted index are also obtained. For this analysis, weekly returns are preferable to daily returns since the former tend to smooth out very short-term fluctuations.

This data base differs from the one used by Greene and Fielitz [12]. They employed five years of daily data containing 1225 observations commencing at the end of 1963. Although the statistical sample sizes are approximately equal, economically the 18.5 year chronological time span seems preferable since it provides a more realistic opportunity for nonperiodic cycles to reveal themselves.

III. Rudiments of R/S Analysis

The primary use of R/S analysis is to detect one variety of long-term dependence. In the R/S sense, positive long-term dependence (persistence) occurs when there is relatively more variability in the low frequencies than in the high ones. White noise exists when the variability is uniform across all frequencies. Negative dependence is present when high frequencies contain more variability than do the low frequencies. All three cases are consistent with the presence of short-term random fluctuations. In fact, it is the purpose of R/S analysis to remove these obscuring short-term patterns so that the long-term behavior may be more readily observed.

Following the lead of Greene and Fielitz [12], the G-Hurst estimator, which was developed by Mandelbrot and Wallis [20], is used. This approach is the one that has been commonly employed in other economic studies and has been demonstrated to be statistically robust.[4] A thorough statistical description of the technique is beyond the scope of this paper; nevertheless, it is necessary to define quantitatively the Hurst coefficient, to describe how it is estimated, and to discuss its interpretation.[5]

The components of R/S analysis may be defined in the following way. Let $X(t)$ be a discrete, stationary time series containing T observations and $X^*(t)$ be the series' corresponding cumulative sum. The range, $R(t,s)$, is given by

$$R(t,s) = \max(d) - \min(d) \quad \text{for } 0 \leq u \leq s, \tag{1}$$

where

$$d = \{X^*(t+u) - [X^*(t) + (u/s)(X^*(t+s) - X^*(t))]\},$$

and where t is any starting point in X^* and s is the length of any feasible subseries. In this formulation, the range reflects the trend of the subseries, which is entirely defined by the subseries' endpoints, t and $t+s$. Schematically, $R(t,s)$ for a given starting point and subseries size is depicted in Figure 1.

4. Other examples of R/S applications concerning price behavior include Booth, Kaen, and Koveos [7], Helms, Kaen, and Rosenman [14], and Booth and Koveos [8].

5. For a comprehensive understanding of the technique, see not only Mandelbrot and Wallis [20] but also Wallis and Matalas [27], and the citations contained in each. Note that other approaches to measure the Hurst coefficient exist. For instance, Hurst originally suggested a point estimate based on the entire series. Also Salas et al. [26] have proposed a local slope estimator, which approximates the Mandelbrot and Wallis measure if the underlying stochastic process is self-similar. Still other approaches exist but they presuppose a particular stochastic process. For example, McLeod and Hipel [21] develop a maximum likelihood estimator that requires fractional Brownian motion.

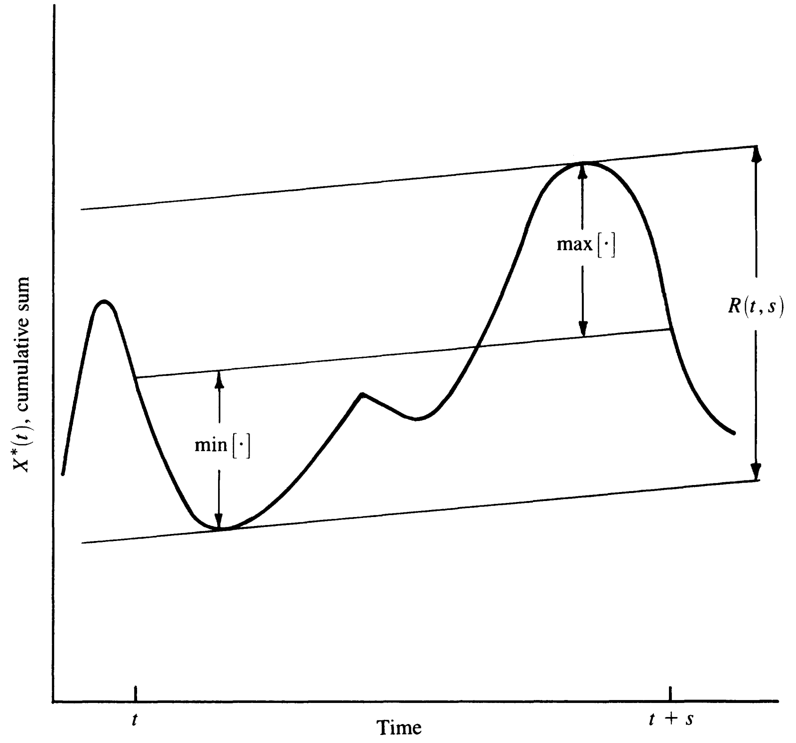

Figure 1. Illustration of $R(t, s)$

If the marginal distribution is non-normal, reliance on $R(t, s)$ to measure the Hurst coefficient may lead to erroneous conclusions. This potential problem, however, is easily remedied by rescaling the range by dividing it by the standard deviation of the original series, $S(t, s)$, where

$$S(t,s) = \{(1/s) \sum_{u=1}^{s} X^2(t+u) - (1/s)^2[\sum_{u=1}^{s} X(t+u)]^2\}^{.5}. \tag{2}$$

Since non-normal distributions are commonplace, this range rescaling is routine.

Mandelbrot and Wallis [20] state that the rescaled range of a subseries is asymptotically related to its length. Specifically,

$$R(t,s)/S(t,s) \sim cs^h, s \geq 3, \tag{3}$$

where c is a scale constant that is partially determined by the original series' volatility and h is the Hurst coefficient.

To estimate h, numerous R/S values are obtained by varying the subseries size. In this case, the G-Hurst (hereafter GH) procedure is used to obtain 38 subseries, ranging in length from $s = 10$ to $s = 890$.[6] The specific values of s are provided by Wallis and Matalas [27] and have been demonstrated by them to extract the maximum amount of information. For each subseries size, 15 samples are selected. Beginning with the initial observation, these samples are sequential and are uniformly spaced throughout the time series. For each of the 15 samples, the R/S statistic is

6. An alternative to this sampling with replacement approach is the F-Hurst procedure. It uses all possible subseries to estimate h rather than only a selected few. Simulations by Wallis and Matalas [27], however, indicate that the G-Hurst procedure provides results very similar to F-Hurst and uses much less computer time.

calculated. The Hurst coefficient is determined by first transforming relation (3) into an equality and linearizing so that

$$\ln[R(t,s)/S(t,s)] = \ln(\hat{c}) + \hat{h}\ln(s) + e, \tag{4}$$

where the dependent variable is the mean of the 15 samples. Then equation (4) is estimated using ordinary least squares regression. Two regression estimates (GH(10) and GH(50), with the number in parentheses denoting the length of the smallest subseries) have become commonplace. Although the GH(50) specification reduces the number of regression observations (in this case from 38 to 24), Mandelbrot and Wallis [20] point out that if there is some evidence of preasymptotic behavior, GH(50) is preferred because the offending transcient observations (i.e., preasymptotic behavior) are more likely to have been eliminated.

From an interpretational perspective, when $\hat{h} = .5$, the series is labeled as containing only white noise. Positive long-term dependence, the Hurst phenomenon, is present if $.5 < \hat{h} < 1.0$. In this case, observations tend to stay away from the overall mean for an extended period of time; they change direction cross and the mean, and then move away from the mean, also for a long period of time. The behavior repeats itself but no regularity in the pattern of directional switching exists. When $0 < \hat{h} < .5$, negative long-term dependence exists. In this instance, observations tend to offset each other, giving the observed series the appearance of short-term irregular cycles. The extreme Hurst values are $h = 0$ and $h = 1$, with the former signifying that the underlying series is a sine wave (periodic cycle) and the latter indicating that the series contains a nonstationary mean in the form of a trend.

One difficulty underlying the Mandelbrot and Wallis [20] technique (and others as well) is that $\hat{h}$ is a biased estimator. In addition, the regression assumption that the error terms are serially independent is routinely violated. Thus the classic t-statistic criterion used in hypothesis testing must be cautiously interpreted and, perhaps, discarded. As a reflection of the theoretical asymptotic relationship between R/S values and subseries size, the magnitude of the bias is related to the length of the overall series. Bias is also related to the nature of underlying stochastic process itself. Despite its importance, work investigating the nature and extent of bias is sparse. Other than Wallis and Matalas [27], there are but a few examples, and these, like Anis and Lloyd [1], usually presuppose different Hurst estimation procedures. Nevertheless, as a result of analyzing different types of stochastic processes, Wallis and Matalas [27] indicate that although on the average the potential bias may not be large, a large variance among the individual estimates exists. Depending on the stochastic process, it appears that for a series containing 1000 observations, the G-Hurst estimates are overstated by values ranging from .01 to .04, with corresponding standard deviations ranging in size from .05 to .10. As a specific example, application of the GH(10) procedure to repeated simulations of an independent Gaussian process, results in 68 percent of the Hurst coefficients falling between .47 and .57 and a median of .51.

In the analysis that follows, specific attention is given to these issues. In particular, the possibility of nonstationary means is examined, and the impact of potential preasymptotic behavior on the Hurst coefficients is explicitly addressed.

IV. The Results

Because of the large number of Hurst coefficients calculated, frequency distributions are used to summarize the estimates. The distributions for GH(10) and GH(50) are displayed in Table I.

Table I. Frequency Distribution of Hurst Coefficients

Hurst Interval	Greene-Fielitz 200 Stocks Total Returns*	200 Stocks Total Returns	200 Stocks Nonsystematic Returns	68 Stocks Total Returns	200 Simulated Stocks Total Returns	60 Simulated Stocks Total Returns
Panel A: GH(10) Hurst Coefficients						
Under .401	—	—	—	—	—	—
.401 – .450	1	—	7	1	7	4
.451 – .500	35	28	40	16	35	15
.501 – .550	93	73	70	24	74	21
.551 – .600	63	73	64	21	69	17
.601 – .650	7	24	18	6	14	2
.651 – .700	1	2	1	—	1	1
Over .700	—	—	—	—	—	—
Note: $\hat{h}$ = .59 for the market index						
Panel B: GH(50) Hurst Coefficients						
Under .401	2	3	3	2	11	5
.401 – .450	26	24	24	7	27	9
.451 – .500	40	41	35	14	37	15
.501 – .550	71	33	39	16	49	15
.551 – .600	40	55	52	17	43	11
.601 – .650	18	30	29	8	23	3
.651 – .700	2	12	13	4	9	1
Over .700	1	2	5	—	1	1
Note: $\hat{h}$ = .57 for the market index						

*[12, 345].

As a first step, consider the first two columns in the GH(10) and GH(50) panels in Table I. In the first column, the frequency distributions for the 200 Hurst coefficients calculated by Greene and Fielitz [12] is reproduced. The second column summarizes the Hurst values for the 200 stocks used in this study. A casual comparison of the respective GH(10) and GH(50) distributions reveals a surface similarity. A χ^2 test, however, indicates that this similarity hypotheses must be rejected in both cases.[7] The mean of this study's distribution is somewhat larger, a result of the right tail containing relatively more observations. This phenomenon tends to confirm the notion that the longer chronological time period covered herein provides a better opportunity for nonperiodic cycles to assert themselves, if they indeed exist. In any case, similar to the Greene and Fielitz [12] results, long-term dependence of some sort does seem to be present in common stock returns.

As indicated earlier, however, it is possible that this phenomenon may be a result of a nonstationary mean or preasymptotic behavior, and consequently, not truly reflect the presence of long-term memory. Turning to the first possibility, an analysis of the autocorrelation functions of each of the 200 stock return series does not reveal any significant autocorrelation patterns nor the

7. For these and all the following χ^2 tests, the following frequency intervals are used: GH(10): .500 and under, .501–.550, .551–.600, and .601 and over; GH(50): .450 and under, .451–.500, .501–.550, .551–.600, and .601 and over. These are obtained by combining some of the categories depicted in Table I in order to provide an adequate number of observations in each cell. Critical χ^2 values at the 95 percent confidence level are 7.81 and 9.49 for GH(10) and GH(50), respectively. The calculated χ^2 values in this case are 21.6 for GH(10) and 59.85 for GH(50).

presence of spikes. These observations support the contention that these distributions are mean stationary.

As an additional check, the market model is used to decompose total return into its systematic and nonsystematic elements.[8] The market is assumed to be appropriately represented by the value-weighted CRSP index. The Hurst coefficients associated with the nonsystematic returns are presented in the third column in Table I. If the market model holds, these coefficients should be compatible with white noise. To test for this relationship, 200 Gaussian independent ($h = .5$) series containing 965 observations are simulated. The frequency distributions for these Hurst coefficients are displayed in the fifth column in Table I. Employing χ^2 tests reveals that the two GH(10) distributions are similar to each other ($\chi^2 = 2.25$) but the two GH(50) distributions are not ($\chi^2 = 13.15$). Thus, from a probabilistic perspective, for GH(10) removal of systematic returns results in white noise residuals. On the average, however, after the removal, some evidence of positive long-term dependence still exists, when measured by GH(50).

Because of the importance of the asymptotic assumptions in estimating the Hurst coefficients, these are checked explicitly by testing whether the relationship between the R/S values and subseries length is proportional (linear in logarithmic form).[9] Operationally, this is accomplished by creating two GH(10) regression models by incorporating a slope dummy in equation (4) that is equal to one for either the last 7 or 15 of the 38 observations and zero elsewhere. If the coefficient of the slope dummy variable is significantly different from zero at the .01 level for both regression tests, the series is classified as displaying preasymptotic behavior. Rescaled range analysis using GH(10) or GH(50) procedures is inappropriate for return series falling into this class. Of the 200 series tested using this approach, only 68 are able to be classified as not exhibiting preasymptotic behavior. The frequency distributions for these Hurst coefficients are reported in the fourth column of Table I. Employing the same testing procedure to the 200 simulated white noise data yields only 60 survivors. The Hurst frequency distribution for these 60 stocks are presented in the sixth column of Table I. Comparison of the 68 stock distributions to their 60 white noise counterparts indicates that the GH(10) distributions are indistinguishable ($\chi^2 = 2.07$) but the GH(50) distributions are somewhat different ($\chi^2 = 11.84$). Similar to the analysis of nonsystematic returns, the GH(50) procedure appears to detect more long-term dependence than the GH(10) procedure.

Finally, from a slightly different perspective, it is useful to note that the mean Hurst coefficients for the simulated white noise series are .538 and .525 for the GH(10) and GH(50) procedures, respectively. Their respective standard deviations are .045 and .078. Moreover, the null hypotheses that these two distributions are normal cannot be rejected (GH(10): $\chi^2 = 2.25$; GH(50): $\chi^2 = 1.84$). These numbers, in conjunction with those mentioned earlier that were developed by Wallis and Matalas [27], suggest that Hurst values between .4 and .6 are commonplace and do not necessarily signal long-term dependence. Thus even the Hurst values for the market returns, which are reported in Table I, may not represent long-term dependence.[10]

8. This approach is an economic adaption of the procedure used by Potter [23] to remove the impact of a potential shifting mean in his study of the presence of long-term dependence in rainfall. Note, however, that it has been recently demonstrated by Chen, Roll, and Ross [9] that in comparison to other macroeconomic factors the market index may not be a particularly important explainer of security returns.

9. This test is similar to using the GH(50) instead of GH(10) estimate of the Hurst coefficient as a measure of long-term dependence. However, not only does it provide an explicit test of the linearity assumption but also it focuses on the R/S behavior of the large subseries.

10. The inability to reach firm conclusions of long-term dependency when $\hat{h}$ lies within the .4 to .6 band has important implications for the Kaen and Rosenman [16] hypothesis. According to Mandelbrot and Wallis [19], the length of the longest nonperiodic cycle is a function of the total number of observations, the value of the Hurst coefficient, and

V. Conclusions

The conclusions from the above analysis are relatively straight-forward. Rescaled Range analysis is a potentially useful technique but conclusions drawn from its application *must* be conditioned on the validity of its underlying assumptions. The apparent pervasiveness of their nonvalidity, however, does restrict the technique's scope.

In the case of common stock returns, the weight of evidence suggests that either long-term dependence is *not* prevalent or that it is *too small* to be accurately measured by rescaled range analysis. A large number of the Hurst coefficients that appear to signify some type of dependence are likely to occur by chance only. The conclusion holds for individual stocks as well as the market. This finding of general nonexistence of long-term dependence in the rescaled range sense, which contradicts the conclusions of Greene and Fielitz [12], suggests that if the information processing behavior as suggested by the competence-difficulty gap hypothesis is to be convincingly accepted, additional empirical support for it must be forthcoming.

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the underlying stochastic process. For instance assuming that the true process is fractional Brownian noise, which they maintain is the best candidate to model the Hurst phenomenon, the ratio between the wavelength and the series length is approximately .33 for $h = .7$ and 0 for $h = .5$. For a 965 week series, $h = .6$ yields a wavelength of over two years. Since h values of less than .6 are not readily distinguishable from white noise, smaller persistent cycles cannot be confidently identified using this technique. Thus, to support their (C-D) gap hypothesis, Kaen and Rosenman [16] need to develop a rationale for at least a two year gap. Their appeal to the learning lag associated with the product development process as a raison d'être for nonperiodic cycles and the C-D gap appears to be misplaced inasmuch as a plausible length for this lag for many products would seem to be less than two years. Exceptions undoubtably do exist, however.

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